Monte Carlo Tree Search for Simultaneous Move Games

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Joint work with Mark Winands[1], Viliam Lisý[2], Christopher Wittlinger[1], Mandy Tak[1]

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December 2nd, 2013

Talk Overview

- Introduction and Background
  - Monte Carlo Search is Everywhere
  - Monte Carlo Tree Search (MCTS)
  - Consistency in Tree Search
- Simultaneous Move Games and MCTS
- Experiments and Results
- Conclusion and Future work

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$\rightarrow$ incl. some new stuff, too!

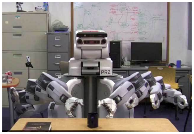
Monte Carlo Search is Everywhere

Coulom ’06, Kocsis & Szepesvári ’06,

Chaslot et al.’07, Gelly et al.’07,

Winands & Björnsson ’08, more..

Van Eyck ’13, Zhu ’13



Lavalle & Kuffner ’01, Perez et al.’11

$$
\begin{aligned}
p(x_0) &= a_0 + x_0(a_1 + x_0(a_2 + \cdots + x_0(a_{n-1} + b_n x_0)\cdots)) \\
&= a_0 + x_0(a_1 + x_0(a_2 + \cdots + x_0(b_{n-1})\cdots)) \\
&\vdots \\
&= a_0 + x_0(b_1) \\
&= b_0.
\end{aligned}
$$

Kuipers et al.’12

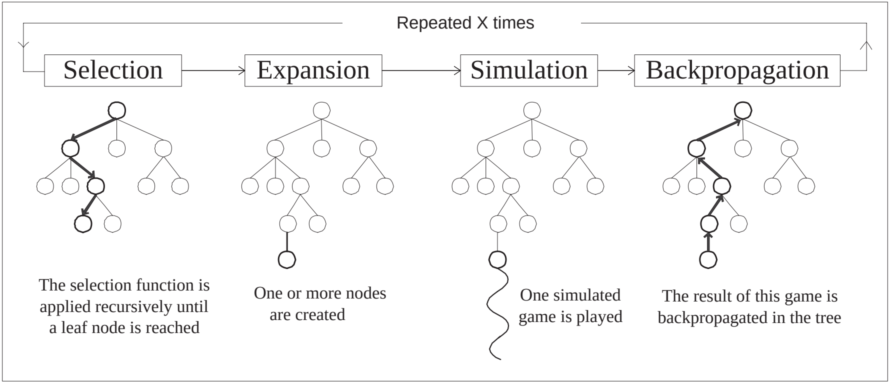
Cazenave ’05, Van den Broek ’09,

Silver & Veness ’10, Ponsen et al.’11,

Cowling et al.’12, Amato et al.’13

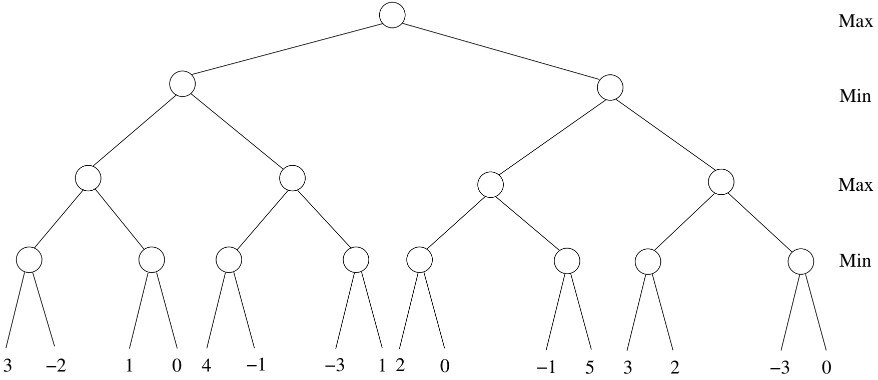
Couetoux et al.’11

Monte Carlo Tree Search: Overview



(Coulom ’06), (Kocsis & Szepesvári ’06), (Chaslot et al.’07), (Browne et al.’12)

Monte Carlo Tree Search (MCTS) Example



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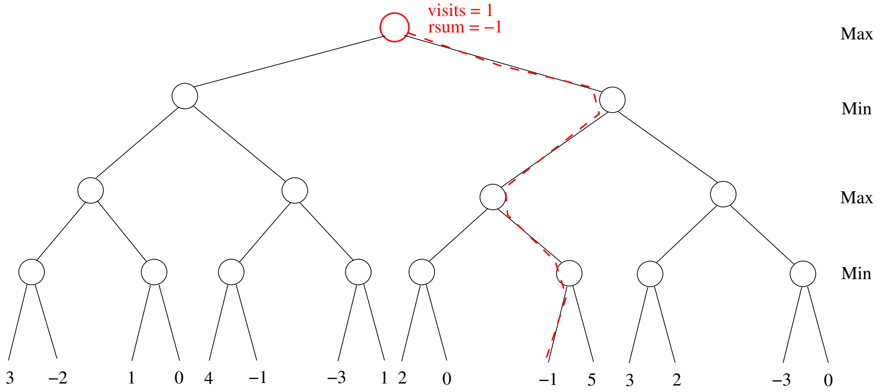

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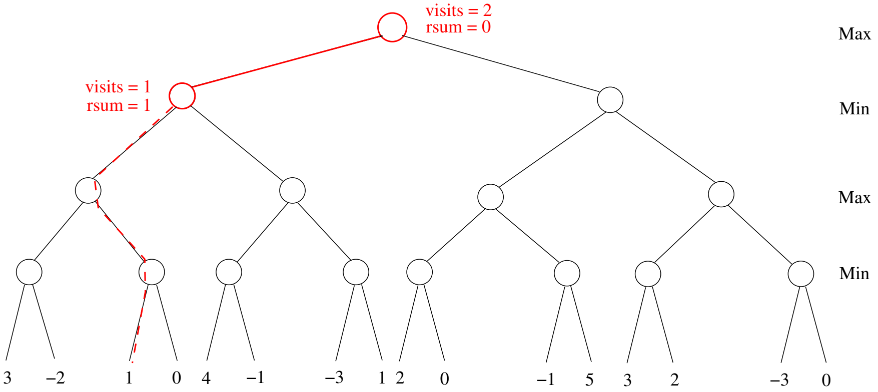

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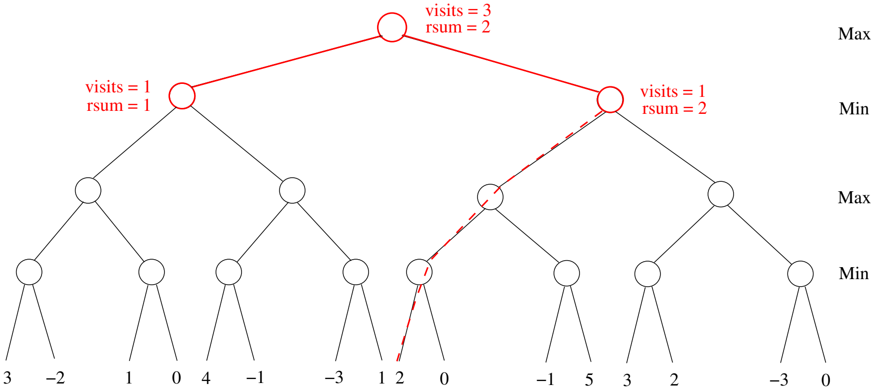

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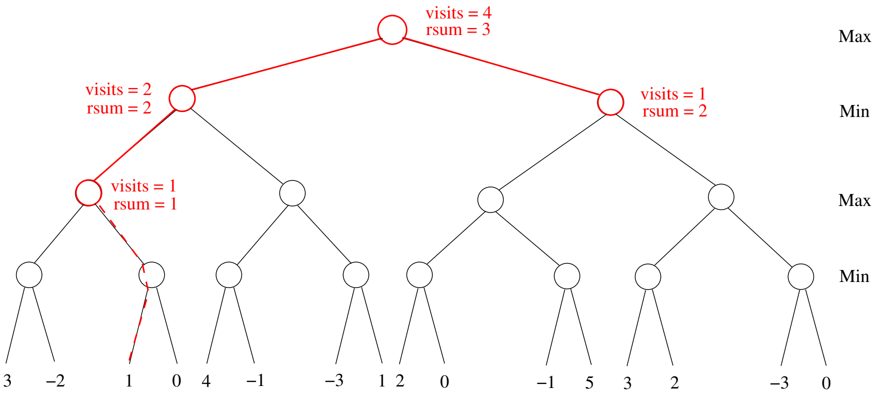

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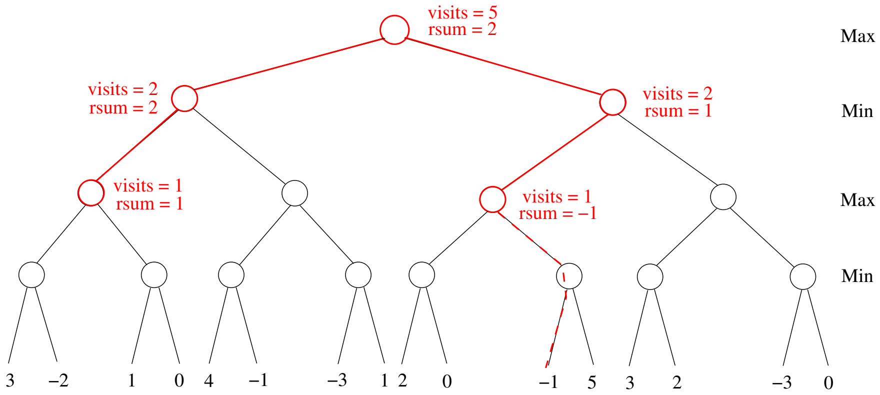

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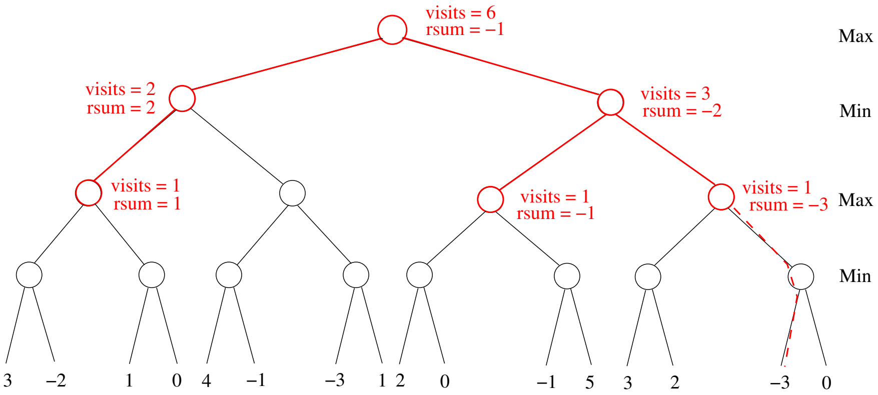

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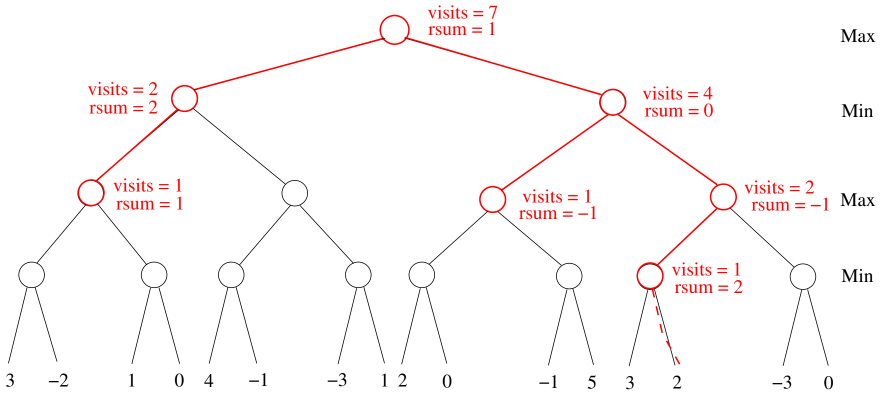

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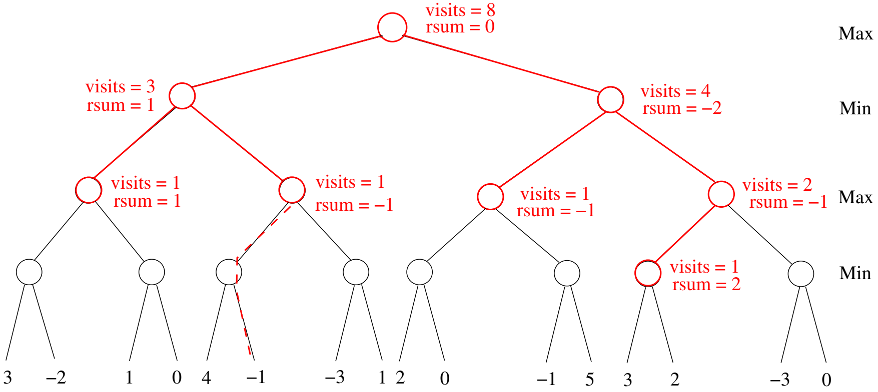
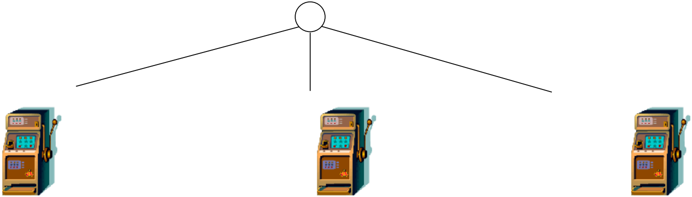


Bandit-Based Selection

To select a node, employ bandit algorithms (i.e. UCB (Auer et al.'02)).



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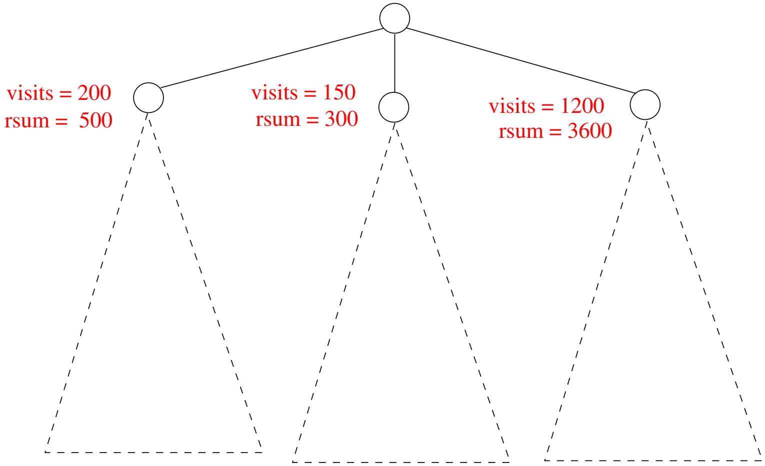


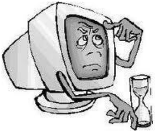
$\Rightarrow$ UCT algorithm (Kocsis & Szepesvári '06).

Consistency in Game Tree Search

Algorithm $A(s,t)$ computes strategy $\sigma(s)$ at state s after t time units.

Computation(search) time...

start computation | - - - - - - t seconds - - - - - - | output strategy



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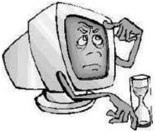

In two-player zero-sum games, for every state s, $\exists$ Nash equilibrium (optimal) strategy $\sigma^*(s)$.

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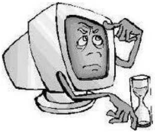

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Definition (Weak/Asymptotic Consistency)

A search algortihm is *(weakly) consistent* if $\forall s, \lim_{t\to\infty} A(s,t) = \sigma^*(s)$.

Consistency in MCTS

Is MCTS consistent?

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- > 2 players?
  Yes, in seq. MP perfect information, using max^n (Sturtevant et. al. '08)

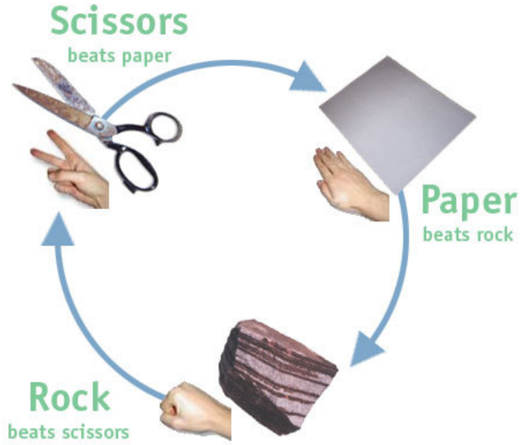
Consistency in MCTS

Is MCTS consistent?

- Yes!
  $\rightarrow$ *in sequential 2P games with perfect information, using UCT*
- > 2 players?
  Yes, in seq. MP perfect information, using $\max^n$ (Sturtevant et. al. '08)
- MCTS-Solver
  - converges faster
  - increases play strength
  - See: (Winands et al.'08, Baier '13, Nijssen '13, more..)

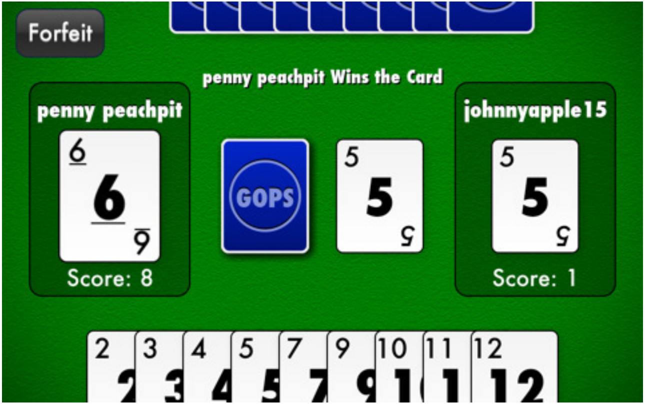
Many optimizations that work better than plain UCT in practice!

Simultaneous Move Games



Each player takes a single action (simultaneously) and game over!

Multiple Stages: Goofspiel



- $(13!)^3 \approx 2.41 \cdot 10^{29}$ unique sequences
- Backward induction, proposed (Ross 1971)
- Solved (Rhoads & Bartholdi 2012)

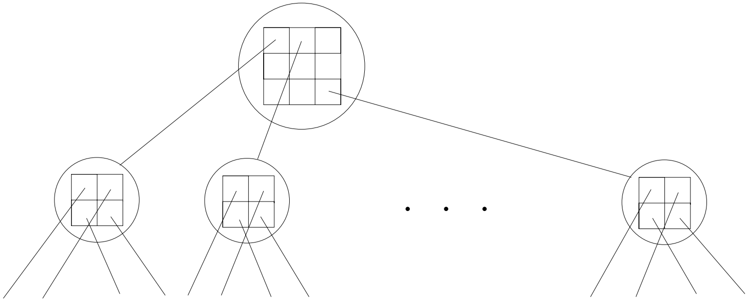
Single Stage: Matrix Game

	r	p	s
R	0.5	0	1
P	1	0.5	0
S	0	1	0.5

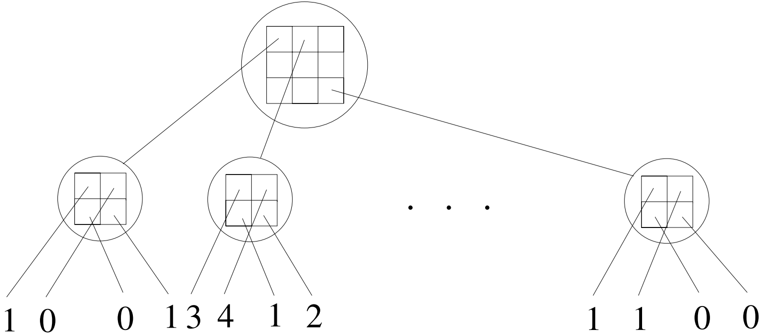
Let $\sigma_{col} = (r, p, s)$. Maximize V such that $r, p, s \geq 0$ and

$$\begin{array}{ccccccc} \frac{1}{2}r & & & + & s & \geq & V \\ r & + & \frac{1}{2}p & & & \geq & V \\ & & p & + & \frac{1}{2}s & \geq & V \\ r & + & p & + & s & = & 0 \end{array}$$

Multi-stage: Simultaneous Move Games



Backward Induction



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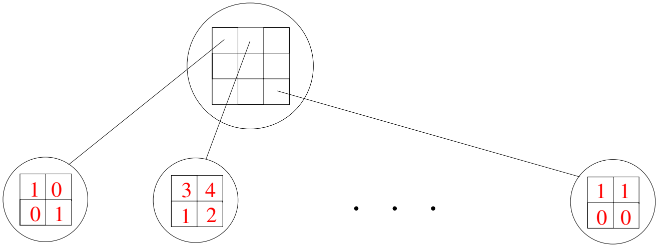

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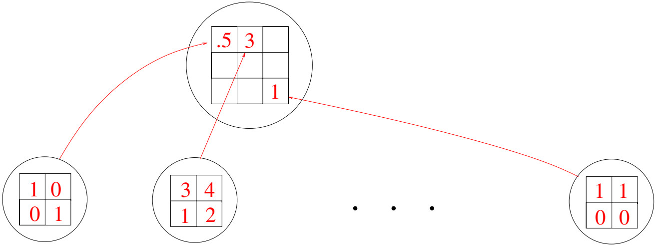

Simultaneous Move Minimax

Extension to classic game-tree search: “SM-Minimax”:

- Depth-first search, as before
- Values sent back up are solutions to LPs
- Finite depth? $\rightarrow$ Evaluation function
- Weakly consistent

Recent work:

- Simultaneous Move $\alpha\beta$ (SMAB), (Saffidine et al.’12)
- Move Pruning in Serialized $\alpha\beta$, (Bosansky et al.’13)

Simultaneous Move MCTS

Extension to MCTS: “SM-MCTS”:

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- Regret minimization in unknown matrix games:

  Stochastic Bandits (UCB) $\rightarrow$ *“Adversarial” Bandits (Exp3, RM)*

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Previous work:

- Backward induction (Ross 1971, Buro '03)
- Sequential UCT (Several 2008 - 2011)
- Decoupled UCT (DUCT) (Finnsson et. al. '08, Perick et. al. '11)
  - Provably inconsistent (Shafiei et. al. '09)
- Pruning in SM-MCTS (Finnsson '12)
- Exp3 at each stage (Auger '11, Teytaud & Flory '11)
  - Both: empirical evidence of consitency

Our SM-MCTS Variants

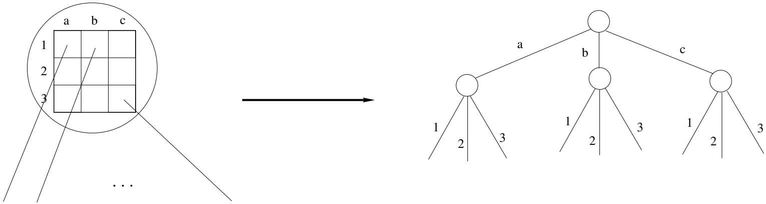
So far, we have looked at:

- Decoupled UCT
- Sequential UCT
- UCB1-Tuned (Sequential and Decoupled)
- Exp3 (Auer et al.'95)
- Regret Matching (RM) (Hart & Mas-Colell '00)
- Online Outcome Sampling (OOS)
- ϵ-minmax

in several domains ...

Sequential UCT

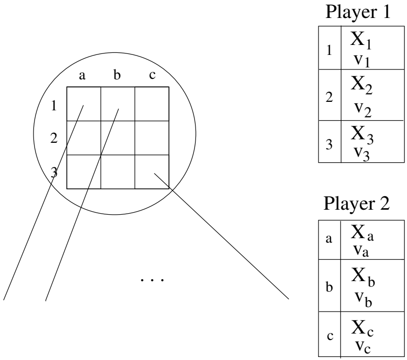
Model the game as a sequential game.



Then apply usual UCT for selection.

Decoupled UCT (DUCT)

Each player i keeps their own reward estimates and visits for $a_i \in \mathcal{A}_i(s)$



Use UCB for selection. When done simulations:

- DUCT(max): choose $\operatorname{argmax}_{a \in \mathcal{A}(s)} X_a / v_a$
- DUCT(mix): choose a with prob $v_a / \sum_{b \in \mathcal{A}_i(s)} v_b$

DUCT Consistency?

In (Shafiei et al. '09),

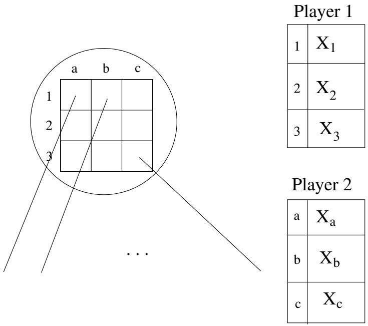
- DUCT(mix) converges to $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ in R, P, S
- In biased R, P, S:

	r	p	s
R	0.5	0.25	1
P	0.75	0.5	0.45
S	0	0.55	0.5

  - DUCT converges to a cycle
  - However, strategy is not a Nash eq.
- Similar no-convergence in Kuhn Poker (Ponsen et al. '11)

Exp3

Each player i keeps their own reward estimates for $a_i \in \mathcal{A}_i(s)$

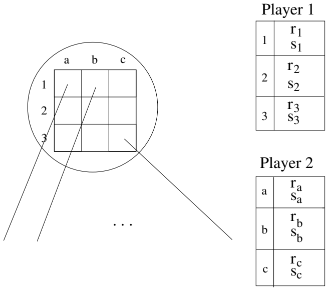


Let $p(a) = \exp(X_a) / \sum_{b \in \mathcal{A}_i(s)} \exp(X_b)$

- Selection: sample a using $\hat{p}(a) = \gamma / |\mathcal{A}_i(s)| + (1 - \gamma) p(a)$
- Obtain reward r from below, update $X_a \leftarrow X_a + r / \hat{p}(a)$
- After sims, choose according to empirical freq. of samples

Regret Matching (RM)

Each player i keeps their own **cumulative regret** for $a_i \in \mathcal{A}_i(s)$



Define $x^+ = \max(0, x)$, and $p(a) = r_a^+ / \sum_{b \in \mathcal{A}_i(s)} r_b^+$

- Selection: sample a using $\hat{p}(a) = \gamma / |\mathcal{A}_i(s)| + (1 - \gamma) p(a)$
- Obtain reward r, accumulate regret for not playing $b \neq a$,
- Add current strategies $p(b)$ to average strategy table s_b for each b.
- After simulations, choose by normalizing s_a for $a \in \mathcal{A}_i(s)$.

ϵ-minmax

After many simulations, we have an approximation for each $X_{a_1 a_2}$.

	a	b	c
1	$\overline{\mathrm{X}}_{1a}$	$\overline{\mathrm{X}}_{1b}$	$\overline{\mathrm{X}}_{1c}$
2	$\overline{\mathrm{X}}_{2a}$	$\overline{\mathrm{X}}_{2b}$	$\overline{\mathrm{X}}_{2c}$
3	$\overline{\mathrm{X}}_{3a}$	$\overline{\mathrm{X}}_{3b}$	$\overline{\mathrm{X}}_{3c}$

- Construct and solve LP to get mixed $\sigma_1(s), \sigma_2(s)$.
- Then, each player selects $a_i \sim \epsilon \cdot \text{Unif}(\mathcal{A}_i(s)) + (1-\epsilon) \cdot \sigma_i(s)$
- MCTS version of MinimaxQ (Littman '98)

Online Outcome Sampling (OOS)

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- Use regret matching over sampled counterfactual regrets
- *Converges to Nash equilibrium over time!*

Win/Loss Goofspiel(13) Performance

P1 \ P2	RND	DUCT(max)	DUCT(mix)	Exp3	OOS	OOS^+
DUCT(max)	76.0					
DUCT(mix)	78.3	57.5				
Exp3	80.0	55.8	48.4			
OOS	73.1	55.3	43.8	47.0		
OOS^+	77.7	67.0	53.3	60.0	57.1	
RM	80.9	63.3	53.2	57.2	58.3	50.4

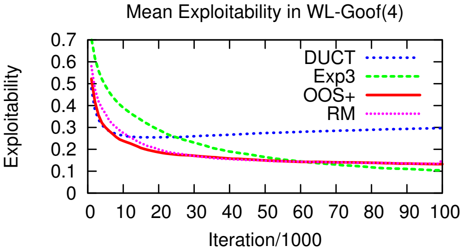
- Number refers to % win rate of row player type.

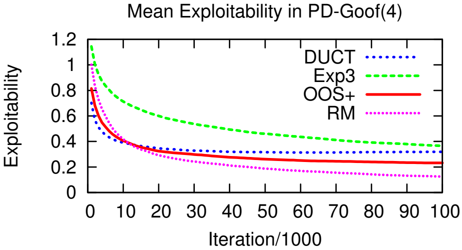
Point Difference Goofspiel(11) Exploitability

Algorithm	Mean Ex_2	Mean Ex_4	Mean simulations per second
DUCT(max)	7.43 ± 0.15	12.87 ± 0.13	124127 ± 286
DUCT(mix)	5.10 ± 0.05	7.96 ± 0.02	124227 ± 286
Exp3	5.77 ± 0.10	10.12 ± 0.08	125165 ± 61
OOS	$\mathbf{4.02} \pm \mathbf{0.06}$	$\mathbf{7.92} \pm \mathbf{0.04}$	186962 ± 361
OOS+	5.59 ± 0.09	9.30 ± 0.08	85940 ± 200
RM	5.56 ± 0.10	9.36 ± 0.07	138284 ± 249

- Lower Ex_d = closer to Nash eq.

WL-Goof(4) and PD-Goof(4) Full Exploitability

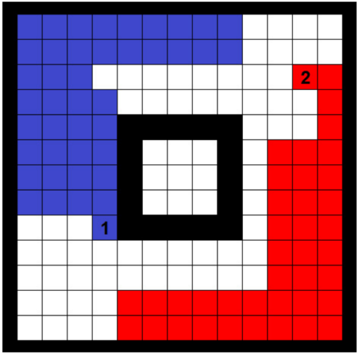




- x-axis is time, y-axis is distance to Nash eq. (lower = closer)

New Results I: Tron

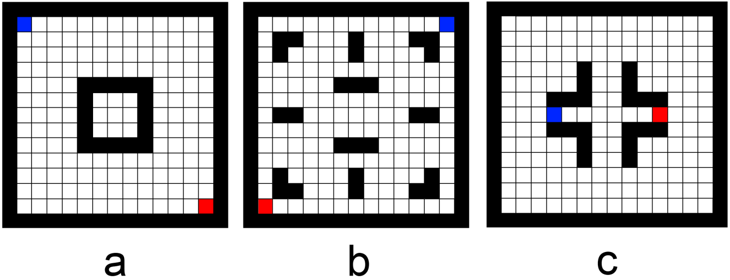
(Lanctot et al.'13). Based on Bachelor thesis of Christopher Wittlinger.



Try to survive and block opponent in a maze.

New Results I: Tron

Three boards below, plus empty board.



Heuristic knowledge in playouts (space estimation + predictive expansion strategies; see also Bachelor thesis of Niek Den Teuling.)

New Results I: Tron

Variant	a	b	c	d	Total
DUCB1T(max)	65%	62%	62%	59%	$62.32 \pm 0.56\%$
DUCB1T(mix)	56%	57%	53%	53%	$54.82 \pm 0.61\%$
UCB1T	58%	57%	49%	54%	$54.32 \pm 0.55\%$
RM	56%	52%	51%	53%	$53.13 \pm 0.62\%$
UCT	47%	54%	55%	49%	$51.39 \pm 0.55\%$
DUCT(max)	43%	54%	49%	50%	$49.05 \pm 0.61\%$
DUCT(mix)	39%	40%	43%	36%	$39.51 \pm 0.64\%$
Exp3	35%	24%	38%	45%	$35.47 \pm 0.61\%$

- Percentage is a win rate
- Results of the different variants played against each other
- $\pm$ refers to 95% confidence intervals.

New Results II: Consistency Guarantees

(Lisý et al.'13) shows that:

- *Any* regret-minimizing alg. leads to weak consistency in SM-MCTS
- Must back-propagate the means for guarantee
- Worst-case analysis of Exp3 and RM on random games

Preliminary Results I

Win percentages of ϵ-minmax in Goofspiel:

vs. DUCT(max)	75.70 %
vs. DUCT(mix)	48.60 %
vs. Exp3	78.50 %
vs. OOS	31.05 %
vs. OOS$^+$	55.75 %
vs. RM	47.55 %

- All are roughly ± 3.0 for 95% c.i.
- Must solve LP only so often
- Must decay ϵ
- Seems more sensitive to parameters

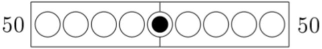
Preliminary Results II

In Tron, run RM with purification: if probability < 0.2, flush it to 0 and renormalize.

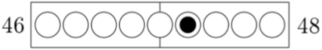
	RM	RM + purification
vs. DUCB1T(max)	31.52 %	35.07 %
vs. DUCT(max)	68.48 %	50.30 %

- RM(purification) wins 54.44% againsts RM
- All are roughly ± 2.0 for 95% c.i.

Preliminary Results III: Oshi-Zumo



[50,4,∗]–Oshi–Zumo starting position – code (50, 50, 0)



Position after move (4,2) – code (46, 48, 1)

Solved in (Buro '03). All results are versus DUCT(max)

	OZ [50,3,1]	OZ [15,3,1]
DUCT(mix)	16.60 %	36.30 %
Exp3	31.10 %	44.95 %
OOS	11.40 %	25.50 %
OOS$^+$	23.85 %	42.40 %
RM	41.80 %	58.60 %

- All are roughly ± 2.5 for 95% c.i.

Conclusions

In Goofspiel:

- Regret matching and OOS$^+$ perform best in Goofspiel
- DUCT(max) performs worst overall
- DUCT(mix) surprisingly good
- OOS converges to Nash in the limit

In Tron:

- DUCB1T(max) is the clear best

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Future work:

- Apply in general game-playing (with Mandy Tak)
- Adaptive algorithms
- Compare to SM-MCTS move pruning (Finnsson '12)
- Compare to SMAB (Saffidine et al. 2012)
- Compare to Serialized Alpha-Beta (Bosansky et al. 2013)
- Extend to fully imperfect information setting

Questions?

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This work is partially funded by the Netherlands Organisation for Scientific Research (NWO).

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