MODELS OF THE COLLECTIVE BEHAVIOUR OF AUTONOMOUS CARS

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MODELS OF THE COLLECTIVE BEHAVIOUR OF AUTONOMOUS CARS

László Z. Varga

Introduction

AUTONOMOUS CAR

How do they plan the route to the destination?

CarTV https://www.youtube.com/watch?v=M1ShJhOqtxA

AUTONOMOUS CARS: BENEFITS

- increased mobility of the elderly and disabled people
- better utilisation of travel time
- finding urban places faster
- increased fuel efficiency
- more efficient traffic flow
- less congestion

LESS CONGESTION, OPTIMAL COLLECTIVE BEHAVIOUR

DW: Das Auto der Zukunft https://youtu.be/RY1WHUfT_zk?t=217

HUMAN DRIVEN VS. AUTONOMOUS

- human driven
  - may follow their habits, although these habits may not be optimal
  - psychologically influenced decision may result in altruistic or panic-like behaviour
  - may not always be aware of the relevant information
- autonomous
  - always follow their designed **rational** preferences
  - wider sensory capabilities (we assume they are correct and do not have e.g. the blind spot of deep learning neural nets)
  - telecommunication technologies to "see" beyond objects and to "see" much farther away
  - real-time data
  - **informed decision**

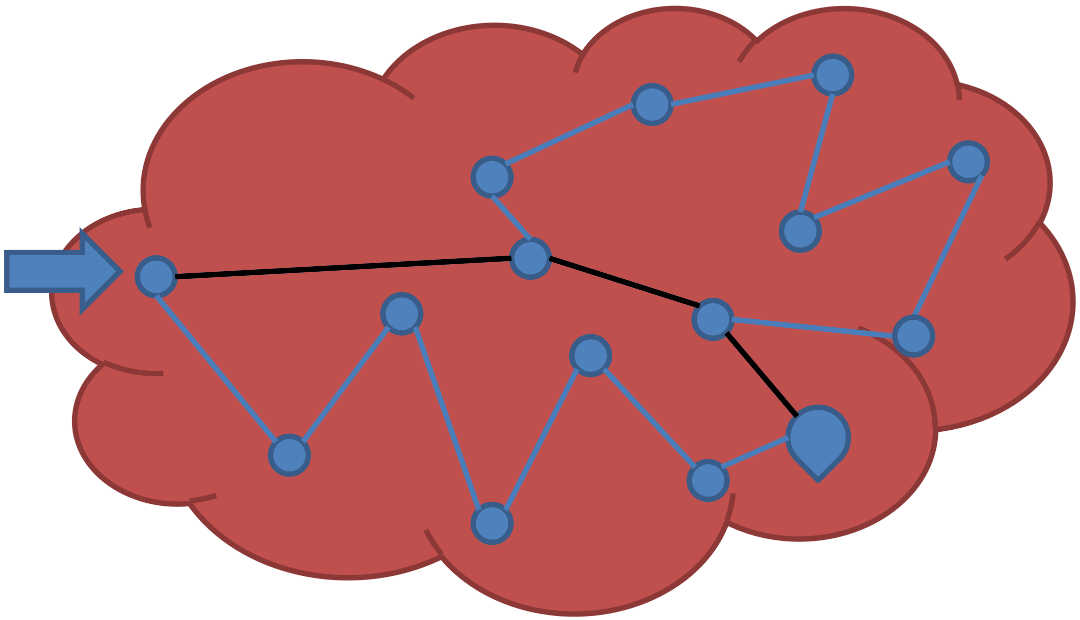
ADAPTATION

- single autonomous car: one actor senses the environment and takes actions to adapt to the changing environment
- several autonomous cars: the overall traffic will emerge as the result of the collective behaviour of several autonomous cars
- can we verify that joint actions of autonomous cars do not generate unwanted behaviour?

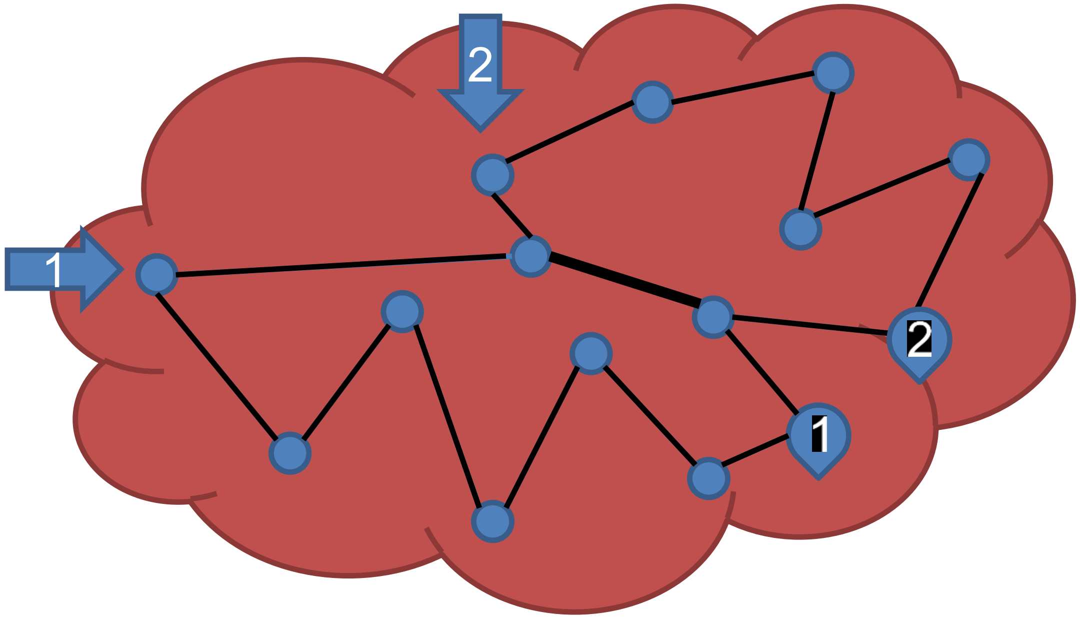
The Routing Problem in the Traffic Engineering Domain

routing problem, preference, optimum, equilibrium, Braess paradox

ROAD TRAFFIC: ROUTING PROBLEM



ROAD TRAFFIC: ROUTING GAME



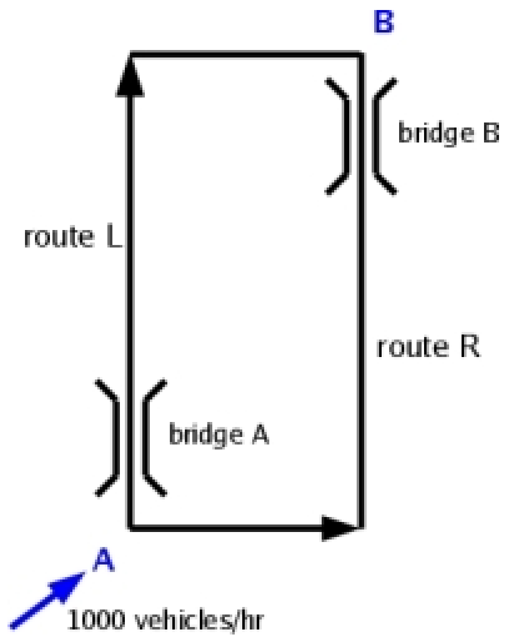
ROAD TRAFFIC: AUTONOMOUS

- a centralized system would be able to create an optimal plan for the trips of the cars
  – optimality: for some "global" parameter
  – fairness: e.g. none of the cars pays with some extra long travel time for the global optimum
- autonomous:
  – traffic participants make autonomous decisions based on their goals and the information available for them locally
- individually self-optimizing travel routes does not necessarily result in optimal traffic

ROAD TRAFFIC: COMPLEX SYSTEM - BRAESS

D. Braess. Über ein paradoxon der verkehrsplanung. Unternehmensforschung, 12:258–268, 1968. link

- road sections are capacious: travel time always takes 15 minutes
- bridges are bottlenecks
- X is the flow rate (number of cars per hour)
- time to cross the bridge is X÷100 minutes



$15 + L \div 100 = 15 + R \div 100$
$L + R = 1000$

- the collective behaviour

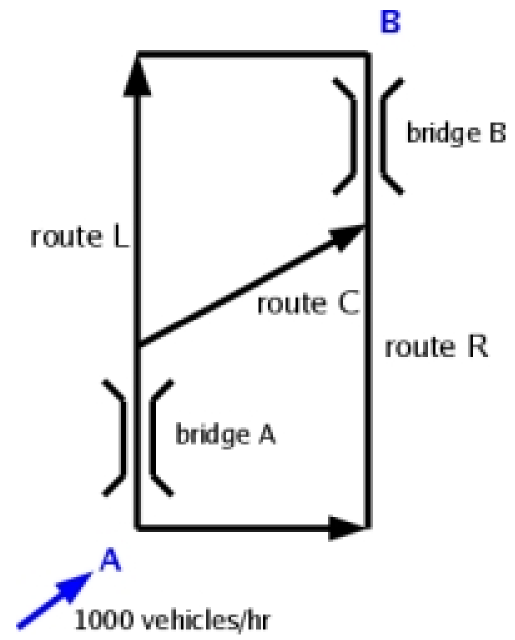
$L = R = 500 \rightarrow 20$ minutes travel

- Nash equilibrium

http://vcp.med.harvard.edu/braess-paradox.html

ROAD TRAFFIC: COMPLEX SYSTEM - BRAESS

- road sections are capacious: travel time always takes 15 minutes
- bridges are bottlenecks
- time to cross the bridge is X÷100 minutes
- central road: always 7.5 minutes



$15 + (L+C) \div 100 = 15 + (R+C) \div 100$
$15 + (L+C) \div 100 = (L+C) \div 100 + 7.5 + (R+C) \div 100$
$L + R + C = 1000$

- the collective behaviour

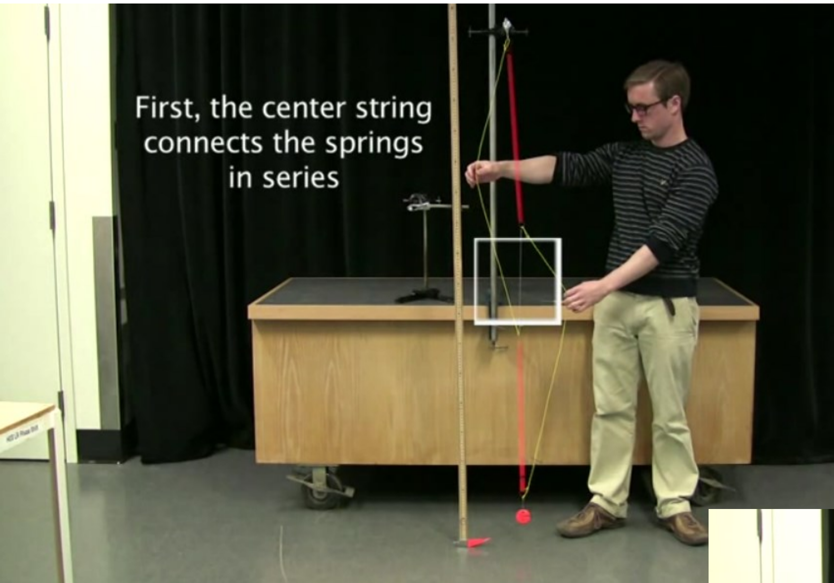
$L = R = 250$, $C=500 \rightarrow 22.5$ minutes travel

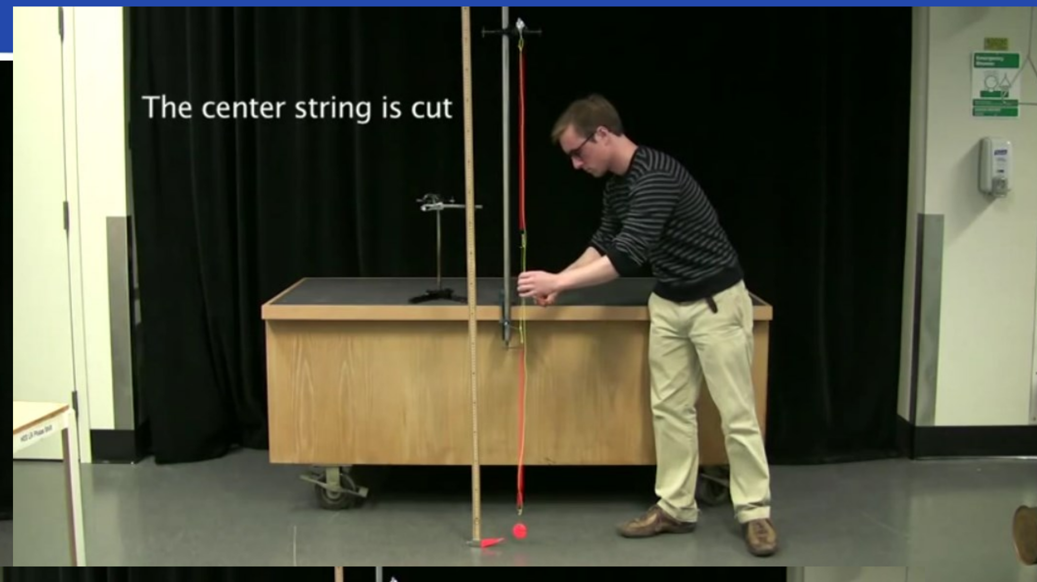
- Nash equilibrium
- they would be better off, if they did not use route C (20 minutes)

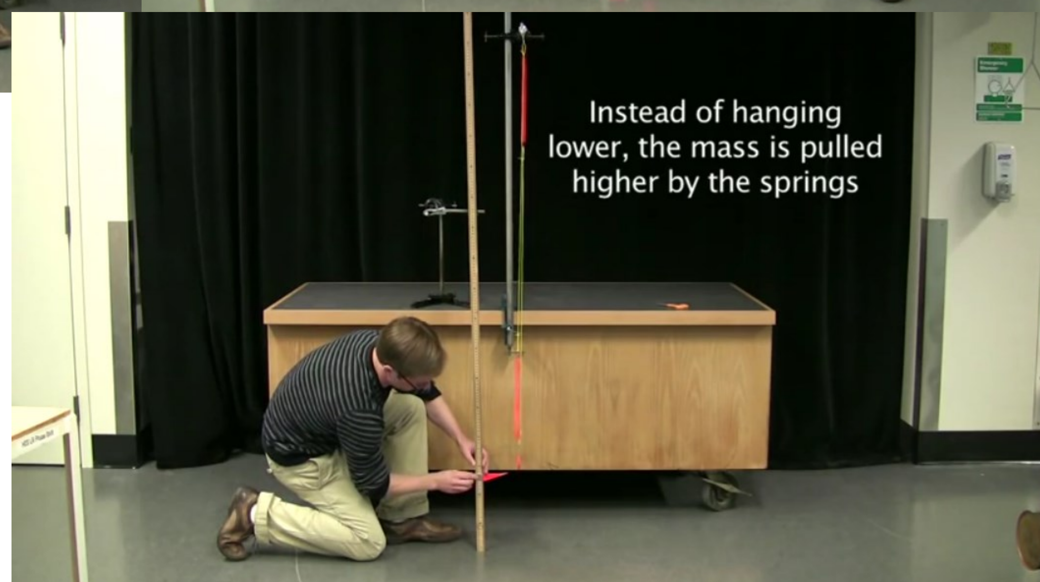
http://vcp.med.harvard.edu/braess-paradox.html

BRAESS PARADOX IN PHYSICS

Joel E. Cohen & Paul Horowitz; Paradoxical Behaviour of Mechanical and Electrical Networks; Nature volume 352, pages 699–701. 1991. link







https://www.youtube.com/watch?v=ekd2MeDBV8s TSG Physics

BRAESS PARADOX IN SOCIAL NETWORKS

Brian Skinner; The Price of Anarchy in Basketball; Journal of Quantitative Analysis in Sports, Volume 6, Issue 1 link

Apt, K.R., Markakis, E. & Simon, S.; Paradoxes in Social Networks with Multiple Products; Synthese (2016); Volume 193, Issue 3, pp 663–687 link

- mid-90s: the New York Knicks seemed to play much better when their superstar centre, Patrick Ewing, was out
  http://blogs.cornell.edu/info2040/2016/09/19/braesss-paradox-in-basketball-the-ewing-theory/

- if satisfaction of the agents is influenced by their neighbours, then
- adding more choices to a node, the network may end up in a situation that is worse for everybody

Classic Game Theory Model

routing game, existence of optimum, price of anarchy, upper bound on the price of anarchy, evolutionary dynamics of repeated games, convergence to the equilibrium

ALGORITHMIC GAME THEORY (ROUTING GAMES)

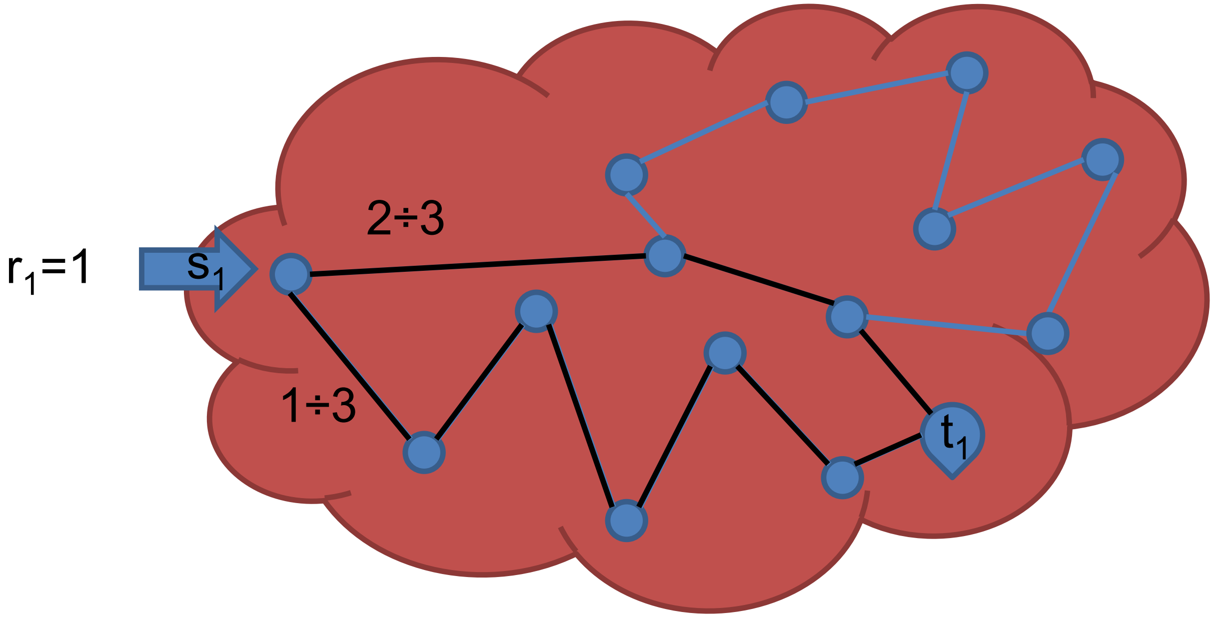
Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay V. Vazirani, Algorithmic Game Theory. Cambridge University Press, New York, NY, USA. 2007 link

chapter: Tim Roughgarden, Routing Games, 461–486.

- directed graph G=(V,E)
- source-sink vertex pairs (s_i,t_i)
- flow r_j for source-sink pairs (a measure of traffic)
- cost function c_e: maps the total flow on the edge to the cost of the edge (continuous, nondecreasing)
- nonatomic routing game: the flow can be divided arbitrarily
  - any rational numbers
  - players for each source-sink pair
  - players decide how to divide their flow among the paths
- atomic routing game: the flow can be divided in units
  - predefined units in rational numbers
  - players for each unit
  - players decide which path to follow

ALGORITHMIC GAME THEORY (ROUTING GAMES)

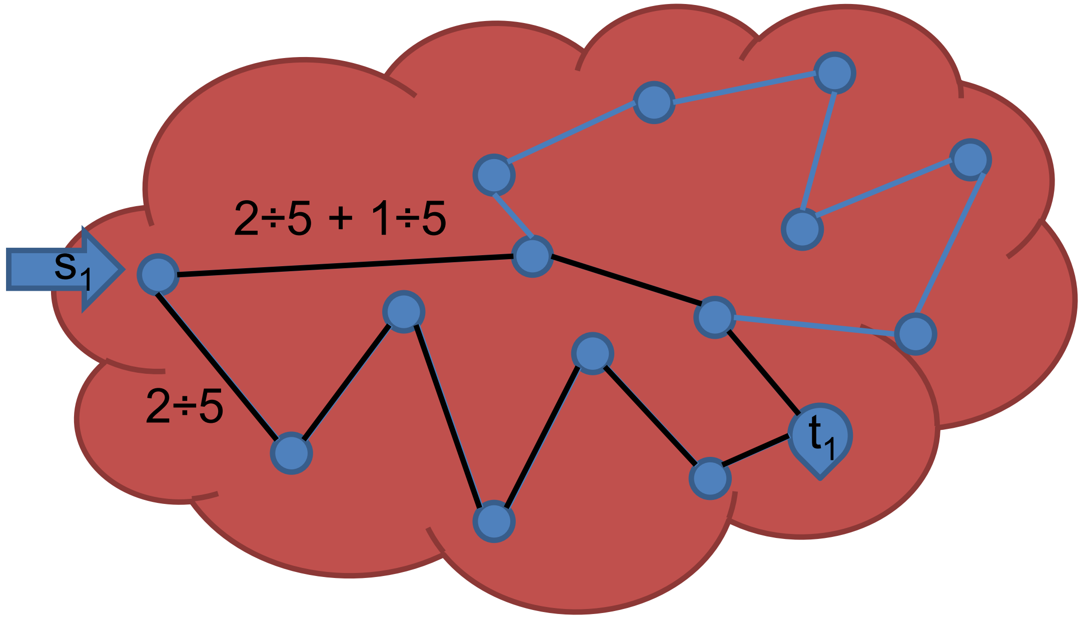
- nonatomic routing game



ALGORITHMIC GAME THEORY (ROUTING GAMES)

- atomic routing game

r_{1a}=2÷5
r_{1b}=2÷5
r_{1c}=1÷5



ALGORITHMIC GAME THEORY (ROUTING GAMES)

- atomic routing game

$r_{1a}=2÷5$
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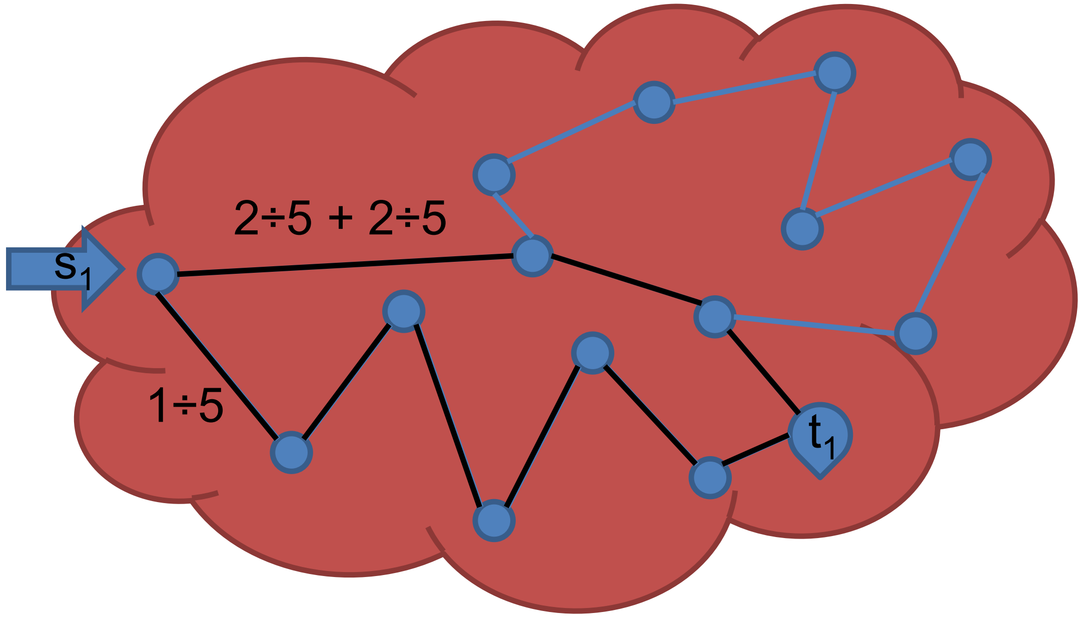
$r_{1c}=1÷5$



ALGORITHMIC GAME THEORY (ROUTING GAMES)

- cost of the flow = sum of the costs of the edges on the path(s)
- cost of the players = sum of the costs of its flow
- selfish players
- equilibrium: none of the players can change its path selection to decrease its cost
- (sum) optimum: the sum of the costs of all players is minimal
- equilibrium > optimum (see the Braess example)
- price of anarchy: the sum of the costs of all players in equilibrium divided by the sum of the costs of all players in optimum

ALGORITHMIC GAME THEORY (ROUTING GAMES)

- nonatomic routing games
  - they have at least one equilibrium
  - all equilibriums have the same cost
- there are atomic routing games that
  - do not have an equilibrium
- there are atomic routing games that
  - have equilibriums with different costs
- if the traffic units of an atomic routing game are equal, then
  - there is at least one equilibrium
- if the cost functions of an atomic routing game are affine (a_e*x+b_e), then
  - there is at least one equilibrium
- the proofs are with a potential function: the change in the player’s cost is identical to the change in the potential function

ALGORITHMIC GAME THEORY (ROUTING GAMES)

- if the cost functions of a nonatomic routing game are affine ($a_e * x + b_e$), then
  - the price of anarchy is at most $4 \div 3$
- if the traffic units (r_i) of an atomic routing game are equal, then
  - the price of anarchy is at most $5 \div 2$
- if the cost functions of an atomic routing game are affine ($a_e * x + b_e$), then
  - the price of anarchy is at most $(3 + \sqrt{5}) \div 2$

EVOLUTIONARY DYNAMICS OF ROUTING GAMES

Simon Fischer & Berthold Vöcking (2004): On the Evolution of Selfish Routing. In: In Proc. of the 12th European Symposium on Algorithms (ESA 04), Springer-Verlag, pp. 323–334. link

- routing game – static
- real life – evolves over time
- agents revise their strategies from time to time based on their observations:
  - the agent observes its own and one of its randomly chosen opponent's payoff
  - and decides to imitate its opponent by adopting its strategy with probability proportional to the payoff difference

EVOLUTIONARY DYNAMICS OF ROUTING GAMES

- Nash equilibrium(s)

	i A	i B
j A	2 , 2	1 , 2
j B	2 , 1	2 , 2

- evolutionarily stable equilibrium

	i cooperates	i defects
j cooperates	3 , 3	1 , 4
j defects	4 , 1	2 , 2

EVOLUTIONARY DYNAMICS OF ROUTING GAMES

- if

  the cost functions of the non-atomic routing game are strictly increasing,

  then the Nash equilibrium is **evolutionarily stable**

- if

  the adaptation probability is the same on all flows of the non-atomic routing game,

  and the initial flow distribution has at least some traffic on each path of the Nash equilibrium,

  then the flow distribution **converges to the Nash equilibrium**

EVOLUTIONARY DYNAMICS OF ROUTING GAMES

- speed of the convergence
  - convergence is defined by the proportion of flow (ε) above the average cost with more than ε (i.e. $1+\varepsilon$ times)
  - time to reach this convergence is logarithmic function of $c_{max} \div c_{avg}$ for single flow non-atomic routing games
  - time to reach this convergence is linear function of $c_{max} \div c_{avg}$ for multi-flow non-atomic routing games

REGRET MINIMISATION IN ROUTING GAMES

Avrim Blum, Eyal Even-Dar & Katrina Ligett (2006): Routing Without Regret: On Convergence to Nash Equilibria of Regret-minimizing Algorithms in Routing Games. In: Proceedings of the Twenty-fifth Annual ACM Symposium on Principles of Distributed Computing, PODC '06, ACM, New York, NY, USA, pp. 45–52. link

- regret over a series of time steps
  - difference between the average latency of the user and the latency of the best fixed path in hindsight for the same origin-destination
- no-regret online algorithm
  - if, for any sequence of flows, the expected regret goes to 0 as the number of steps goes to infinity
  - i.e. the algorithm „learns the best choice"
  - learning algorithms usually need more information (e.g. following the best player) than e.g. playing against a randomly selected player

REGRET MINIMISATION IN ROUTING GAMES

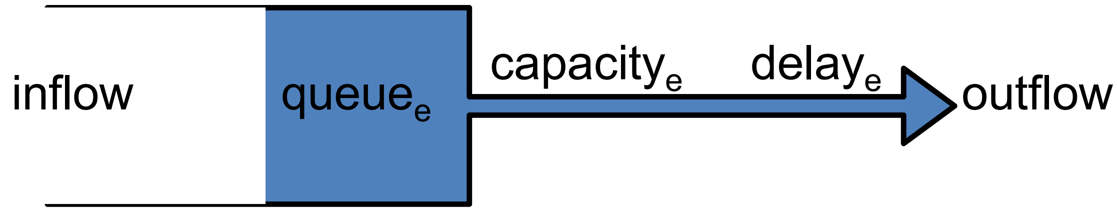
- ε-Nash equilibrium of a flow
  - if the average cost under this flow is within ε of the minimum cost paths under this flow
  - i.e. most users take a nearly-cheapest path
- if each player in a non-atomic routing game uses a no-regret strategy, then the behaviour will converge to ε-Nash equilibrium
  - different bounds on the time needed to reach ε-Nash equilibrium

Dynamic Routing - Queuing Model

traffic flows over time, queuing model, dynamic equilibrium

QUEUING MODEL

- an approximation to investigate how traffic flows evolve over time (the game is not repeated)
- each edge consists of a queue followed by a link which has a constant delay and a maximum capacity



- the cost of the edge is the waiting time in the queue plus the constant delay
- speed of the growth of the queue of the edge is proportional to the difference between the inflow to the edge and the maximum capacity of the edge
- a player controls one flow particle and chooses a source-target-path in the network

NON-ATOMIC QUEUING MODEL – DYNAMIC EQUILIBRIUM AND STEADY STATE

Roberto Cominetti, José Correa, Neil Olver (2017): Long Term Behavior of Dynamic Equilibria in Fluid Queuing Networks. In: Eisenbrand F., Koenemann J. (eds) Integer Programming and Combinatorial Optimization. IPCO 2017. Lecture Notes in Computer Science, vol 10328. pp 161-172. Springer. link

- dynamic shortest path
  - a particle entering the source node s at time θ follows a path $p = e_1 e_2 \cdot\cdot\cdot e_k$
  - it will reach the end of the path at time $T_p(\theta) = T_{e_k} \circ \cdot\cdot\cdot \circ T_{e_2} \circ T_{e_1}(\theta)$
  - the set of paths at time θ with minimal $T_p(\theta)$
- dynamic equilibrium
  - a flow pattern that uses only dynamic shortest paths
- steady state
  - if the cumulative inflows and outflows are equal on all edges all the time

NON-ATOMIC QUEUING MODEL – DYNAMIC EQUILIBRIUM AND STEADY STATE

- single-source-sink flow
- the inflow is not above the total capacity of the network
- proved:
  - the dynamic equilibrium is a steady state
  - a steady state is a dynamic equilibrium
  - a steady state exists and it is actually reached in finite time

- (note: some proofs are „delayed to the full version of the paper”)

NON-ATOMIC QUEUING MODEL – CURRENTLY SHORTEST PATHS

Ronald Koch, Martin Skutella (2011): Nash Equilibria and the Price of Anarchy for Flows over Time. Theory of Computing Systems, July 2011, Vol. 49, Issue 1, pp. 71–97. link

- single-source-sink flow
- Nash equilibrium over time (dynamic equilibrium by Cominetti et al.)
  - a flow pattern that uses only currently shortest paths (dynamic shortest paths by Cominetti et al.)
- the following statements are equivalent
  - flow is only sent along currently shortest paths
  - flow over time is a Nash flow over time
  - no flow overtakes any other flow
- the snapshots at every moment is seen as a static flow and then they prove price of anarchy properties

ATOMIC QUEUING MODEL – FIFO

Martin Hoefer, Vahab S. Mirrokni, Heiko Röglin, Shang-Hua Teng (2011): Competitive Routing over Time. Theoretical Computer Science Volume 412, Issue 39, September 2011, Pages 5420-5432. link

- different coordination mechanisms are defined
- we are interested in FIFO, where tasks are processed in order of arrival
- players have weights w_i
- the processing time on an edge e for agent i is $a_e * w_i * \Delta\tau$
  (i.e. $a_e * w_i$ time steps are needed = constant for FIFO)
- the cost of an edge is the waiting time in the FIFO queue plus the processing time

ATOMIC QUEUING MODEL – SINGLE-SOURCE

- for unweighted single-source temporal network congestion games with the FIFO policy a **Nash equilibrium always exists**
- in every unweighted single-source temporal network congestion game with the FIFO policy **it takes at most n rounds to reach an equilibrium**. In particular, the random and concurrent greedy best-response dynamics reach a strong equilibrium in expected polynomial time

ATOMIC QUEUING MODEL – GENERAL

- there are weighted single-source temporal congestion games with the FIFO policy and **without Nash equilibria**
- there are unweighted temporal congestion games with the FIFO policy and **without Nash equilibria**

ATOMIC QUEUING MODEL – TIME-DEPENDENT COSTS

- cost (i.e. quality of service) depends on the number of players allocating the edge at a given point in time
  - an edge e in the network is fixed to a constant delay d_e (time steps needed)
  - if an edge e is shared at time τ by $n_e(\tau)$ players, all these players get charged cost $c_e(n_e(\tau))$
  - cost incurred by player i on a path $P_i=(e_1,\ldots,e_l)$ is $c_i(\tau_1)=\sum_{j=1}^{l}\sum_{\tau=\tau_j}^{\tau_j+d_{e_j}-1} c_{e_j}(n_{e_j}(\tau))$ where $\tau_j=\sum_{k=1}^{j-1} d_{ek}$

ATOMIC QUEUING MODEL – TIME-DEPENDENT COSTS

- it turns out (according to the paper), that
- if new resources are created for each edge and each time step to reach the given edge, then this is a standard congestion game
- therefore there is a pure Nash equilibrium in every game, and any better-response dynamics converges to Nash equilibrium
- however, the congestion game obtained by this reduction might have a large number of resources, and, in addition, the game is not necessarily a network congestion game
- therefore, the complexity results known for standard network congestion games do not carry over

QUEUING MODEL - EVALUATION

- the queuing model
  - does not have usage dependent cost of the edges if the queue is empty and the inflow is below the maximum capacity, because in this flow range the edge has a constant delay
  - the queuing model has a kind of usage dependent cost of the edges only when the inflow exceeds the maximum capacity of the edges
  - however, above the maximum-capacity flow, the queue grows to infinity over time at constant inflow
- therefore the queuing model is not a complete extension of the static routing game to the time dimension
- in addition, the queuing model assumes that the agents have complete knowledge of the current and future state of all queues and edges

Traffic Generated by Autonomous Cars

real-time traffic information, supporting software applications, online routing problem, evolutionary dynamics in the online routing problem

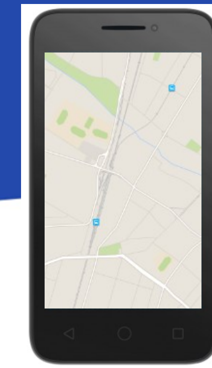
EQUILIBRIUM

John Glen Wardrop (1952): Some Theoretical Aspects of Road Traffic Research. Proceedings of the Institution of Civil Engineers, Part II 1(36), pp. 352–378. link

Martin J. Beckmann, C. B. McGuire, and Christopher B. Winsten. Studies in the Economics of Transportation. Yale University Press, 1956. link

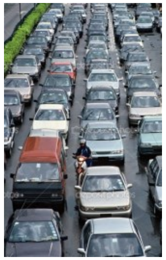
- traffic engineers assume: the traffic is always assigned in accordance with the nonatomic equilibrium
- game theory assumes: agents have complete knowledge about the game, and the agents come to the equilibrium with full rationality
- evolutionary dynamics: the agents receive feedback
  - routing game converges to the equilibrium
- game dynamics assumptions: the decision making is on the flow level; the game is repeated
  - **not realistic for autonomous cars**: the decision making is done at the individual car level, and the decision is based on the real-time situation

ONLINE REAL-TIME DATA



- are there online navigation devices in this room?
  - potentially many,
  - with online data,
  - more and more integrated with cars.

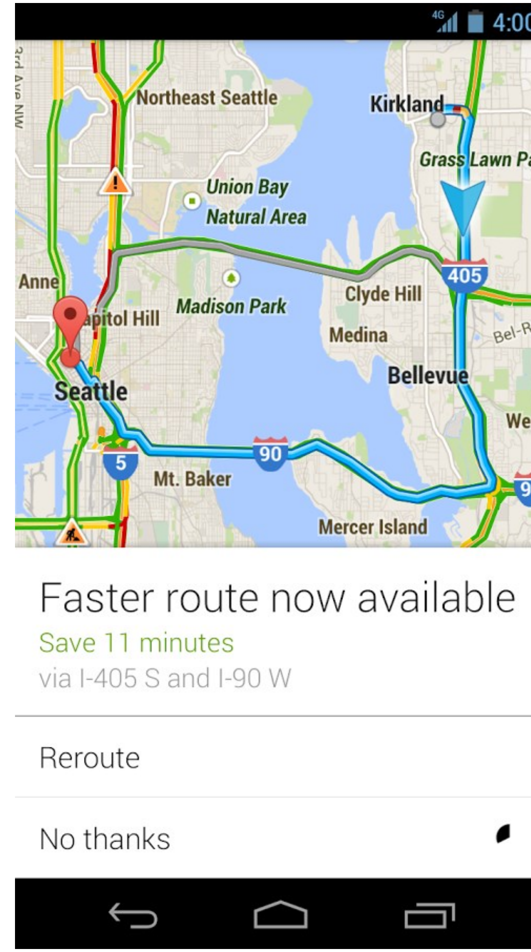
- will the traffic as a whole benefit if traffic is guided by these devices?
  - some say: yes!
  - some say: no!



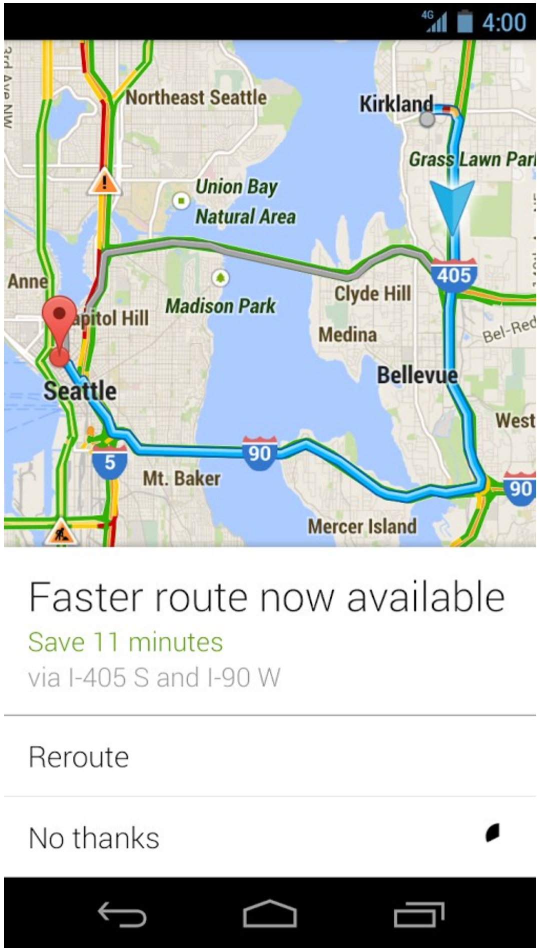
- Wardrop model: the traffic is allocated in accordance with the equilibrium
  - is it still valid with this technical progress?
- we need a theoretical model to study these questions

ONLINE REAL-TIME DATA

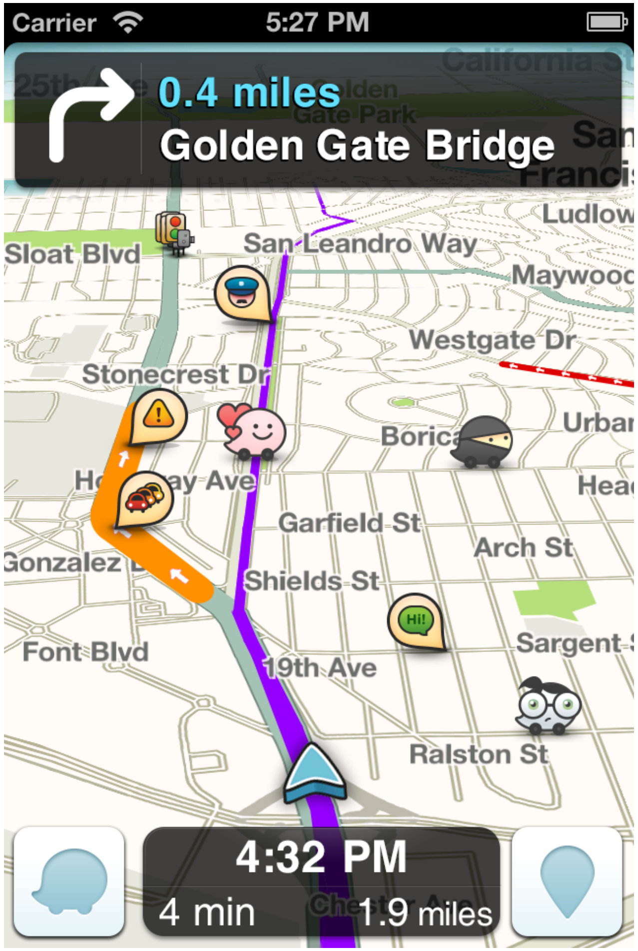
Budapest néhány pontján fényújság tájékoztatja az autósokat az egyes hidak elérhetőségéről. Egy okostelefonnal még jobb, még részletesebb dugóinformációt kaphatunk folyamatosan (Fotó: bkk.hu)



ONLINE REAL-TIME DATA



it is widely believed that route planning results in shorter travel time if we take into account the real-time traffic information



ONLINE REAL-TIME DATA

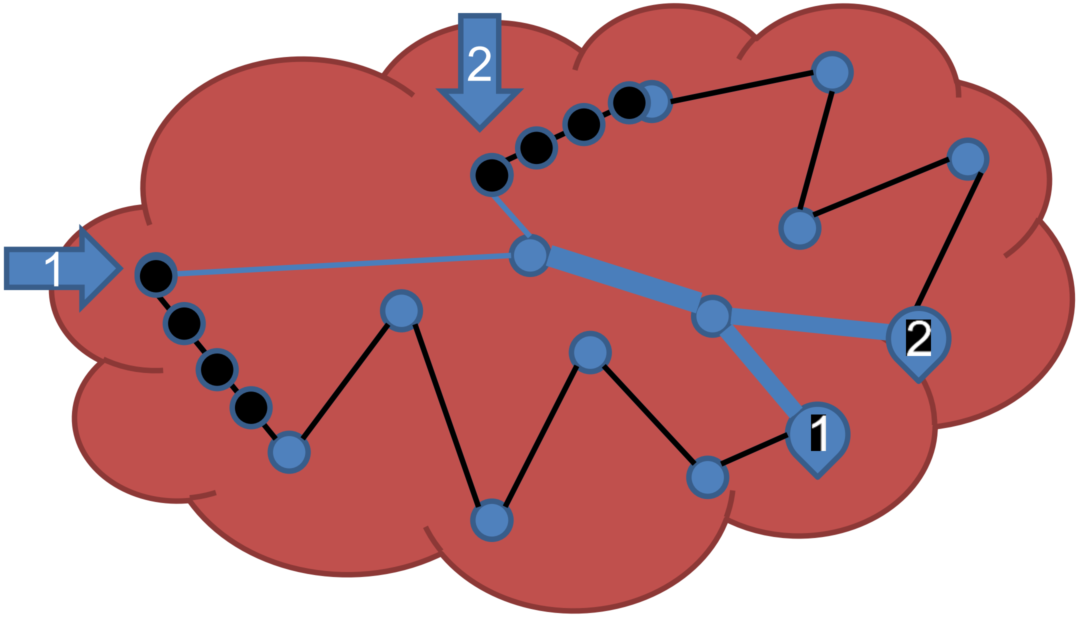
- agents are embedded in their environment
- they perceive the current state
  and
  make decisions which action to perform
  in order to
  adapt to the environment
- the actions are both
  reactive
  and
  proactive

ONLINE REAL-TIME DATA

- traffic networks are complex systems, not only because of the complexity of the road network,
  but also because the traffic changes with delay in response to the actions of the agents participating in the traffic:
  when an agent selects a route to follow, then the agent may contribute to a congestion which will develop in the network sometime later

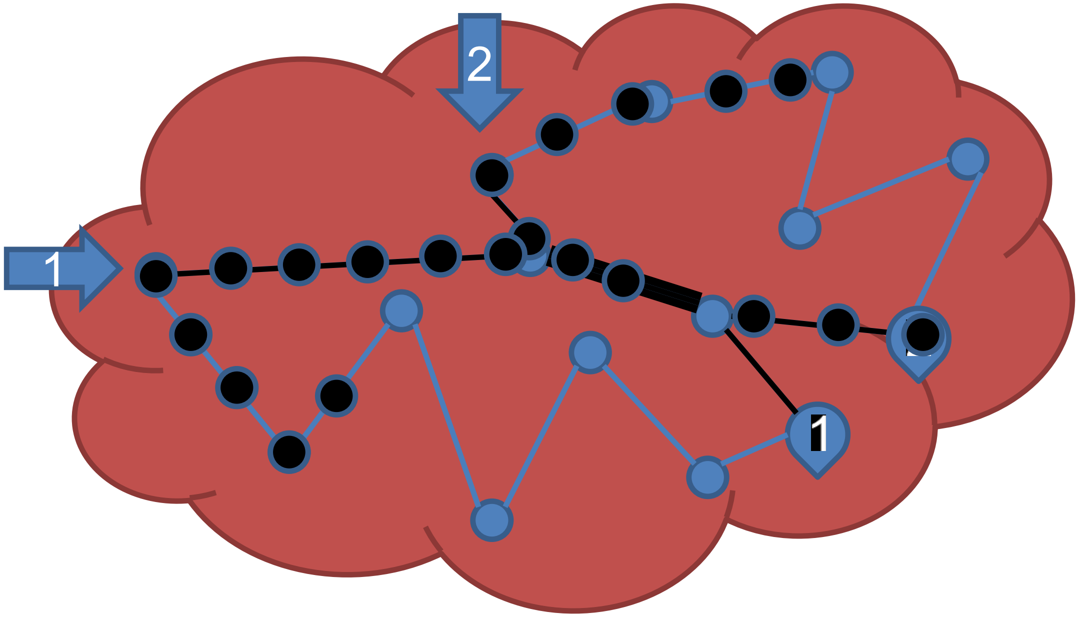
ONLINE REAL-TIME DATA

- decision is based on current situation, which may change while they are on route



ONLINE REAL-TIME DATA

- subsequent agents of the same flow may select different routes

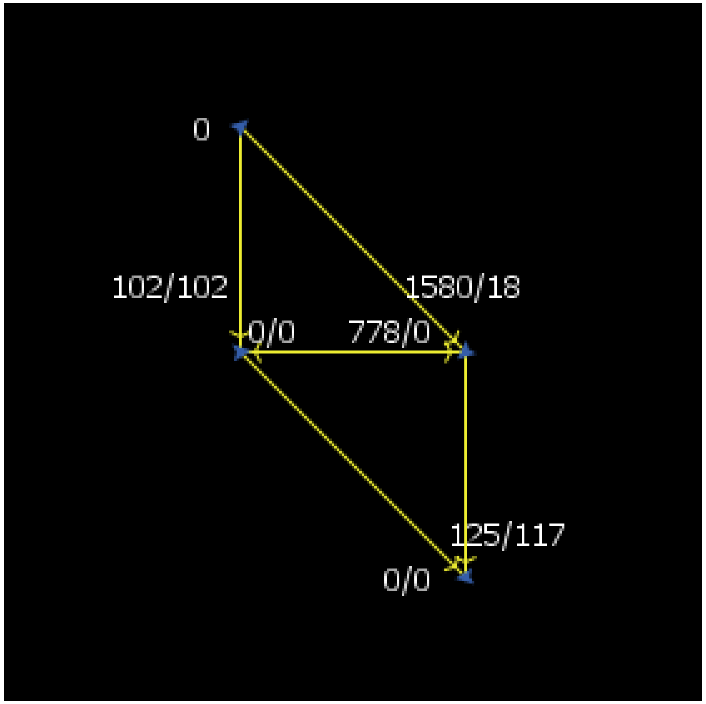


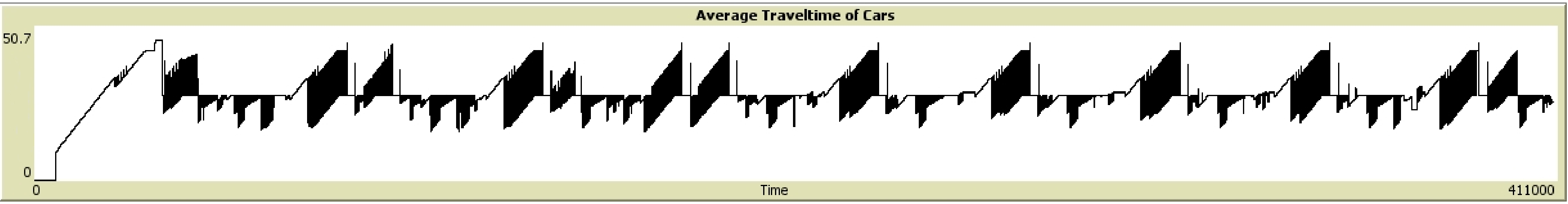
ONLINE REAL-TIME DATA AND PLANNING

- agents are dynamically arriving and departing
- plans are created by exploiting online data that describe the current cost of the resources
- uncertainty about the feasible decision of an agent, because the cost of the resources will change by the time the agent starts to use them:
  - departing agents will release the resources
  - agents simultaneously creating their plans will influence each other's costs
  - agents arriving later may also influence the costs of the resources already used by agents

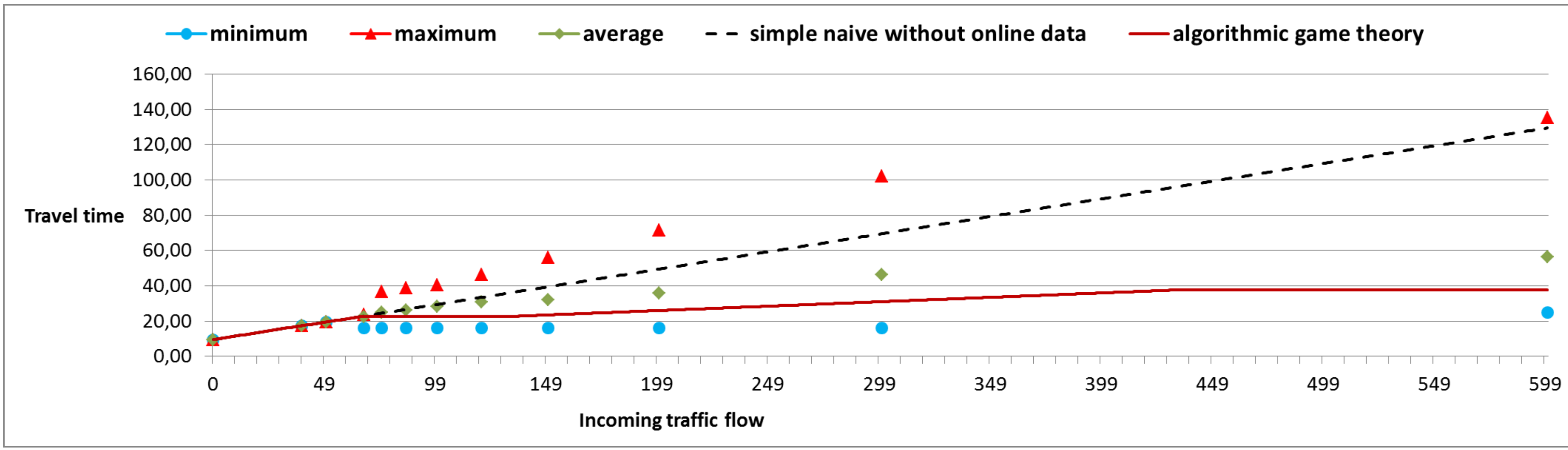
SIMULATIONS

- shortest current travel time strategy in a Braess network





SIMULATIONS



Online Routing Game Model

online routing game, the issue of equilibrium, benefit of online data, intention awareness, benefit in intention aware online routing games

ONLINE DATA: BEYOND ROUTING GAMES

- the throughput characteristic of the network changes with time and the agents do not know this characteristic
- the agents continuously enter the road network and decide their optimal route only when they enter the road network and the decision is based on the live information on the current situation of the road network
- the outcome travel time for a given agent depends on the trip schedule of other agents that entered the network previously, are currently entering the network, or will enter the network later

ONLINE ROUTING GAMES

László Z. Varga: Online Routing Games and the Benefit of Online Data, in Proc. Eighth International Workshop on Agents in Traffic and Transportation, at 13th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2014), May 5-6, 2014, Paris, France, pp. 88-95. link

- sextuple (t, T, G, c, r, k), where
  - t={1, 2, ...}: sequence of equal time periods;
  - T: time periods giving one time unit;
  - G: a directed graph G=(V, E)
  - c: cost function of G with $c_e:R^+ \rightarrow R^+$
  - r: total flow, r_i: aiming for a trip P_i from s_i to t_i
  - $k=(k_1, k_2,\ldots)$: sequence of decision vectors with decision vector $k^t=(k^t_1, k^t_2,\ldots)$ made in time period t and decision k^t_i made by the agent of the flow r_i in time period t

ONLINE ROUTING GAMES

- the cost function maps the flow $f_e(\tau)$ (that enters the edge e at time τ) to the travel time on the edge
- $f_e(\tau)$ is the number of agents that entered the edge e between $\tau-T$ (inclusive) and τ (non-inclusive)
- the cost for the agent entering the edge e is never less than the remaining cost of any other agent already utilizing that edge increased with a time gap gap_e, (FIFO, maximum capacity)
- if two agents enter an edge exactly at the same time τ, then one of them (randomly selected) suffers a delay gap_e, which is part of its cost on edge e, and its remaining cost is determined at the delayed time, so its cost on edge e will be $gap_e+c_e(f_e(\tau+gap_e))$

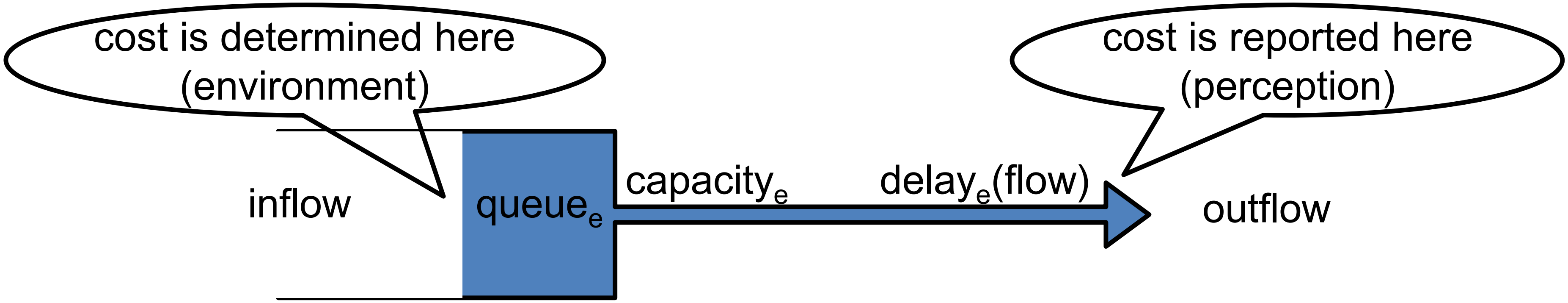
ONLINE ROUTING GAMES

- the variable part of the cost functions are not known to any of the agents of the model
- and the agents can learn the actual cost only when an agent exits an edge and reports it
- the actual cost of a path (e_1, e_2, e_3, …) for a flow starting at time period t is

  $c_{e1}(f_{e1}(t))+$
  $c_{e2}(f_{e2}(t+c_{e1}(f_{e1}(t))))+$
  $c_{e3}(f_{e3}(t+c_{e1}(f_{e1}(t))+c_{e2}(f_{e2}(t+c_{e1}(f_{e1}(t))))))+\ldots,$

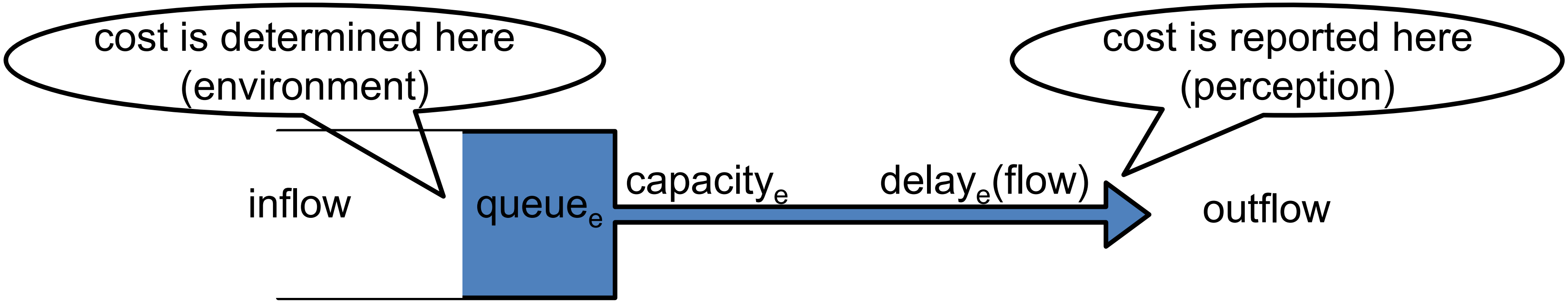
  i.e. the actual cost of an edge is determined at the time when the flow enters the edge

ONLINE ROUTING GAMES



- a complete extension of the routing game to the time dimension
- each edge consists of a queue followed by a link which has a congestion sensitive delay and a maximum capacity
- speed of the growth of the queue of the edge is proportional to the difference between the inflow to the edge and the maximum capacity of the edge (i.e. gap$_e$)

ONLINE ROUTING GAMES



- the cost of the edge is the waiting time in the queue plus the delay
- agents interested in the (estimated!) cost values must decrease the last reported value by taking into account the time elapsed since the last reporting event
- a player controls one flow particle and chooses a source-target-path in the network
- **resembles to online navigation software available for autonomous cars**

SIMPLE NAÏVE STRATEGY

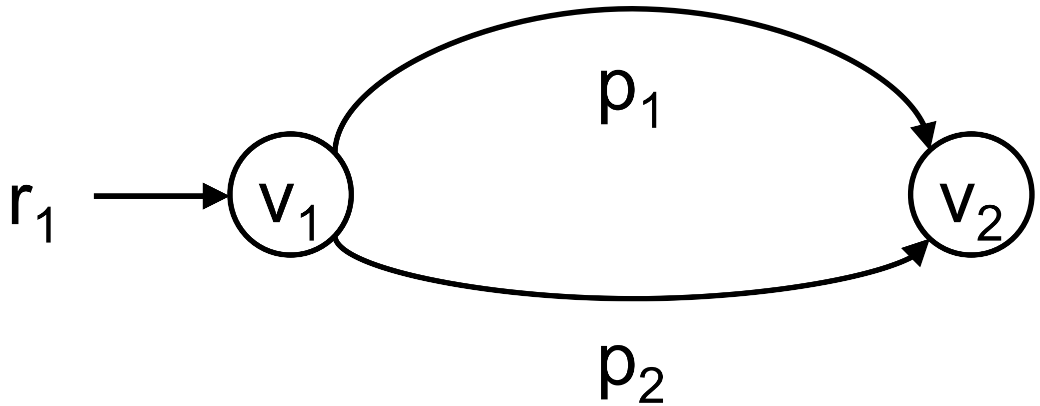
- the decision k^t_i is how the trip P_i is routed on a single path among the paths leading from s_i to t_i
- we are investigating how basic navigation systems perform in online routing games
- typical navigation software use shortest travel time search
- we call this decision strategy as *simple naïve strategy*

BENEFIT OF ONLINE DATA

- *Definition 1.* The *worst case benefit* of online real-time data at a given flow is the ratio between the cost of the maximum cost of the flow and the cost of the same flow with an oracle using the same decision making strategy for only the fixed part of the cost functions.
- *Definition 2.* The *best case benefit* of …
- *Definition 3.* The *average case benefit* of …

EQUILIBRIUM IS NOT GUARANTEED

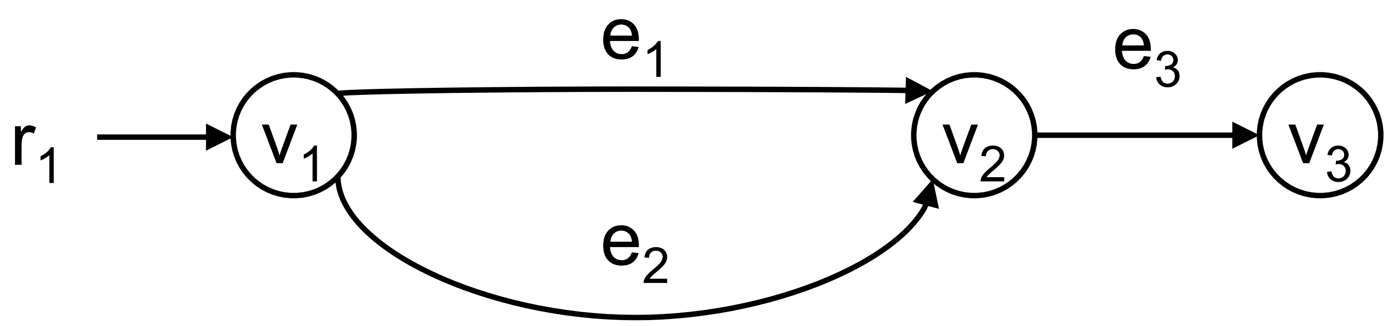
- THEOREM 1. *There are simple naïve strategy online routing games which do not have equilibrium at certain flow values.*



$c_{c1} > c_{c2} > c_{nc1} > c_{nc2}$

SINGLE FLOW INTENSIFICATION

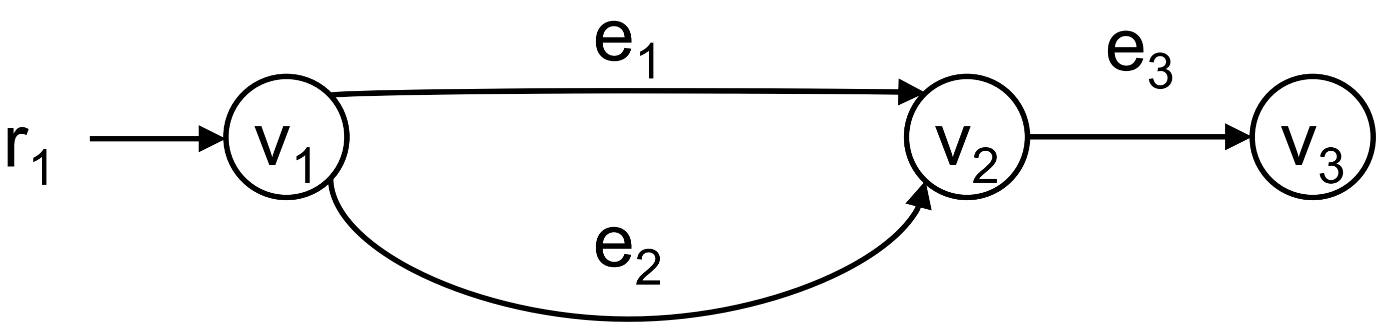
- THEOREM 2. *There are SN=(t, T, G, r, c, k) simple naïve strategy online routing games where the total traffic flow has only one incoming flow, i.e.* $r=(r_1)$*, however the flow on some of the edges of G sometimes may be more than* r_1.



$c_2>2$ $1.5*c_2>c_{c1}>c_2>c_{nc1}$

WORST CASE BENEFIT IS NOT GUARANTEED

- THEOREM 3. *There are SN=(t, T, G, r, c, k) simple naïve strategy online routing games where the worst case benefit of online real-time data is greater than 1, i.e. in these games the worst case benefit is a "price".*



$c_2 > 2 \quad 1.5 * c_2 > c_{c1} > c_2 > c_{nc1} \quad c_{c3+} > c_{c3}$

intention awareness

INTENTION PROPAGATION

R. Claes, T. Holvoet, and D. Weyns: A Decentralized Approach for Anticipatory Vehicle Routing Using Delegate Multi-agent Systems, in IEEE Transactions on Intelligent Transportation Systems, vol. 12, no. 2, pp. 364-373, 2011. link

- anticipatory vehicle routing
  - a vehicle agent running on a smart device inside the vehicle
  - vehicle agents communicate their individual planned route to the delegate MAS
  - the delegate MAS makes forecast of future traffic density
  - the delegate MAS sends back the traffic forecast to the vehicle agents which use this information to plan their trip

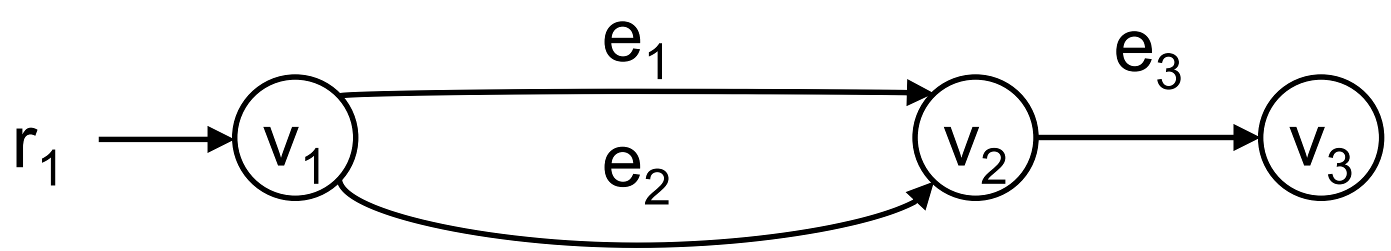
SNIP ONLINE ROUTING GAMES

László Z. Varga. On Intention-Propagation-Based Prediction in Autonomously Self-adapting Navigation. Scalable Computing: Practice and Experience, 16(3):221–232, 2015. link

- simple naive intention propagation online routing games are online routing games where
  - the decision making agents of the flows are the vehicle agents
  - the delegate MAS predicts the travel times for each path of the trip
  - the decision is to select the path with the shortest predicted travel time

SINGLE FLOW INTENSIFICATION

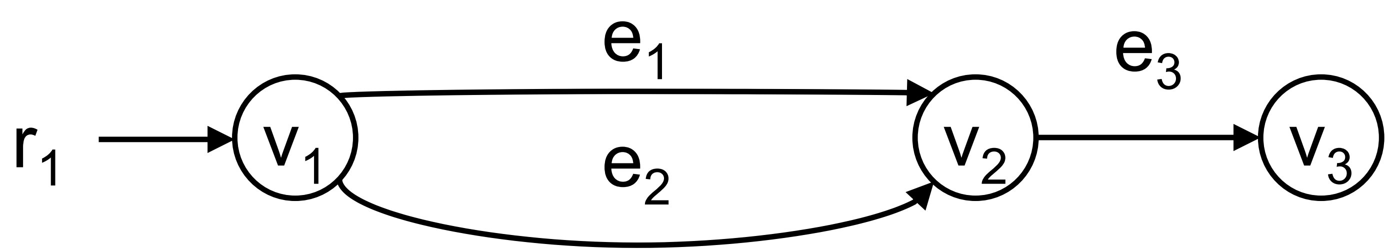
- THEOREM 1. *There are simple naïve intention propagation strategy online routing games where the total traffic flow has only one incoming flow, i.e.* $r=(r_1)$*, however the flow on some of the edges of G sometimes may be more than* r_1*.*



$C_{e_1}=10+x \quad C_{e_2}=10.5+x \quad C_{e_3}=1+x \quad r_1=1$

SINGLE FLOW INTENSIFICATION

- PHENOMENON 1. *If we increase the throughput capacity between v_1 and v_2 with a parallel road e_2, then the traffic on e_3 will fluctuate and will be sometimes worse than without e_2.*



$C_{e_1}=10+x \quad C_{e_2}=10.5+x \quad C_{e_3}=1+x \quad r_1=1$

WORST CASE BENEFIT IS NOT GUARANTEED

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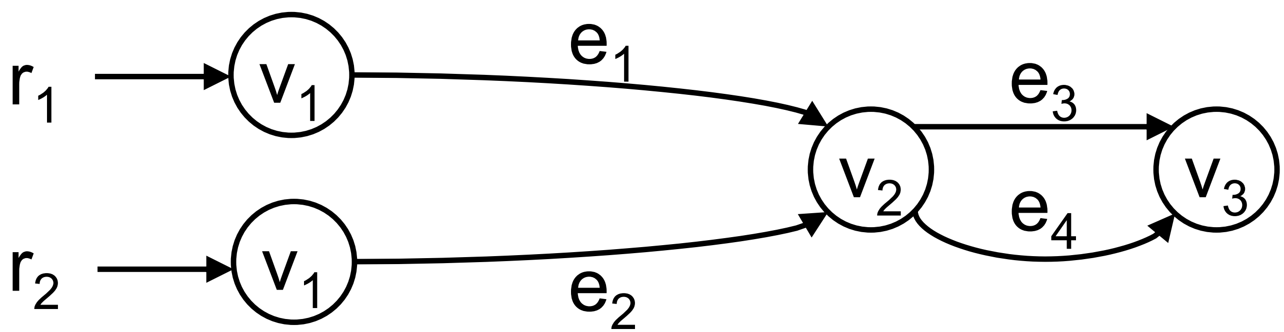
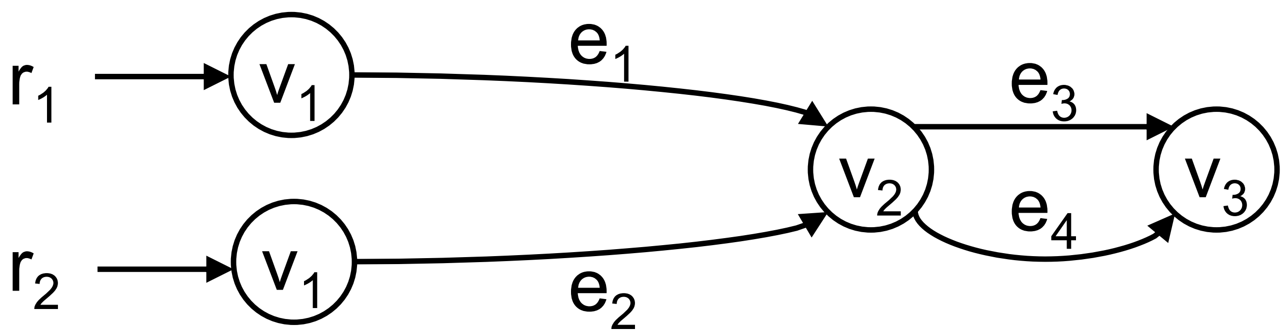


$C_{e_1}=1$ $C_{e_2}=1$ $C_{e_3}=10+x$ $C_{e_4}=10.5+10*x$ $r_1=r_2=1$

WORST CASE BENEFIT IS NOT GUARANTEED

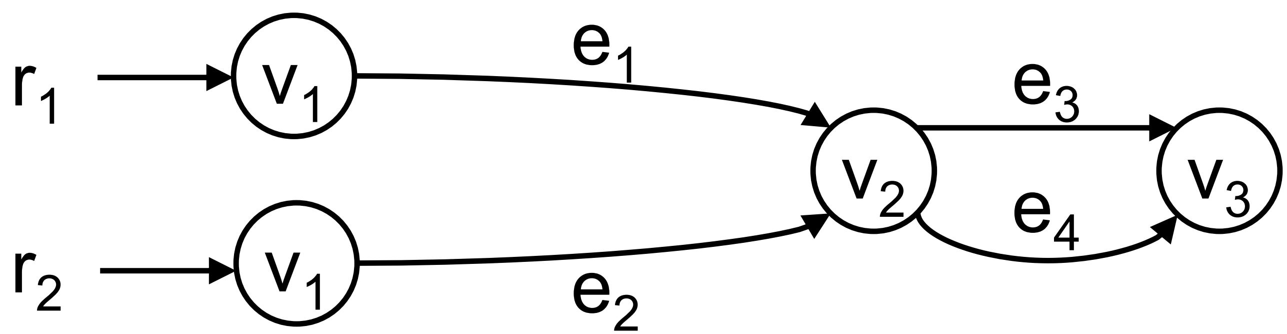
- PHENOMENON 2. *If we increase the throughput capacity between v_2 and v_3 with a parallel road e_4, then the traffic in the whole network will be sometimes worse than without e_4.*



$C_{e_1}=1 \quad C_{e_2}=1 \quad C_{e_3}=10+x \quad C_{e_4}=10.5+10*x \quad r_1=r_2=1$

EQUILIBRIUM IS NOT GUARANTEED

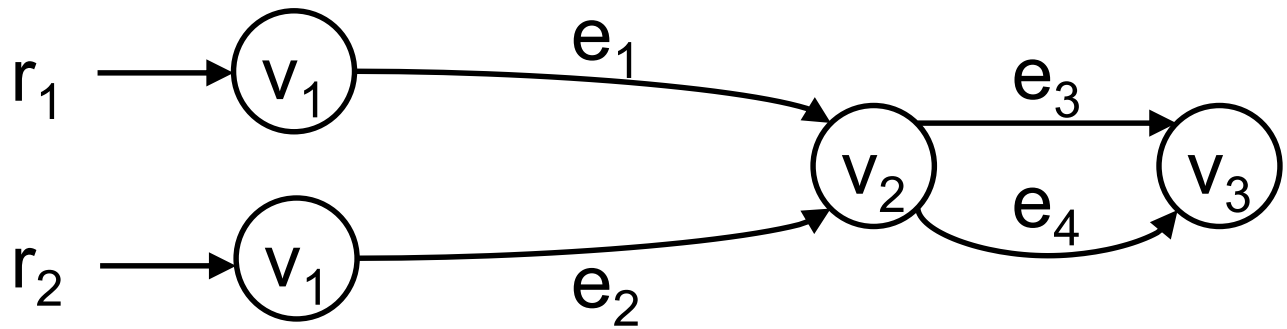
- THEOREM 3. *There are simple naïve intention propagation strategy online routing games which do not have equilibrium at certain flow values.*



$C_{e_1}=1 \quad C_{e_2}=1 \quad C_{e_3}=10+x \quad C_{e_4}=10.5+10*x \quad r_1=r_2=1$

EQUILIBRIUM IS NOT GUARANTEED

- PHENOMENON 3. *If we increase the throughput capacity between v_2 and v_3 with a parallel road e_4, then the traffic in the whole network will fluctuate.*



$C_{e_1}=1$ $C_{e_2}=1$ $C_{e_3}=10+x$ $C_{e_4}=10.5+10*x$ $r_1=r_2=1$

IMPLICATIONS FOR ROAD NETWORK DESIGN

László Z Varga: Paradox Phenomena in Autonomously Self-Adapting Navigation; Cybernetics and Information Technologies 15:(5) pp. 78-87. (2015) link

- two novel forms of paradox phenomena of the online routing game problem, similar to the Braess paradox:
  - If the road network is extended with parallel roads as in some networks, and the cars use online navigation devices, then
  - although the throughput capacity of the network is extended, sometimes the overall performance will be reduced,
  - because at some flow values the traffic might start to fluctuate
- implications for the structure of the road network
  - try to avoid such parallel roads if the cars use navigation devices exploiting online information

CONCLUSIONS

- a theoretical model for the metric and analysis of autonomous navigation based on online real-time data
- we have shown that autonomous self-adaptation with these strategies
  - sometimes make the system fluctuate,
  - sometimes some agents pay a price for the autonomous self-adaptation of the whole system,
  - intention-propagation-based prediction is a form of cooperation, which helps to solve these problems, but not completely
- made recommendation for traffic design
- other forms of cooperative intelligent road transport support systems might be needed

guaranteed benefit in intention aware online routing games

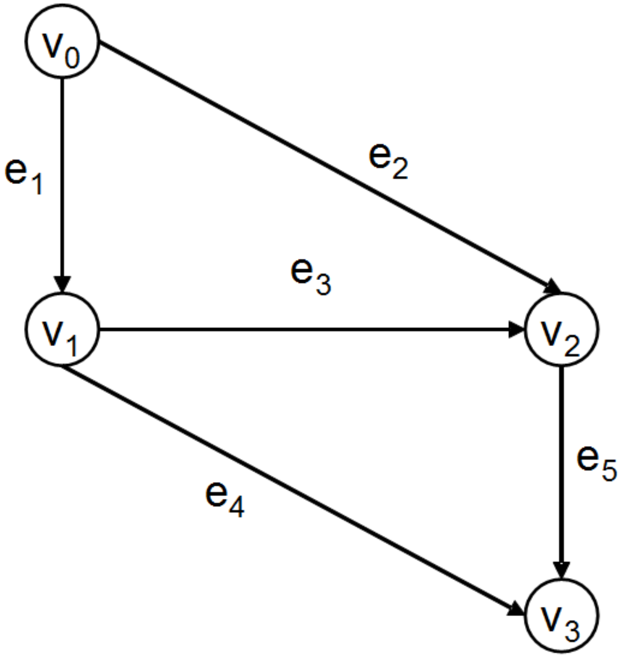
GUARANTEED BENEFIT OF ONLINE REAL-TIME DATA IN THE ONLINE BRAESS PARADOX

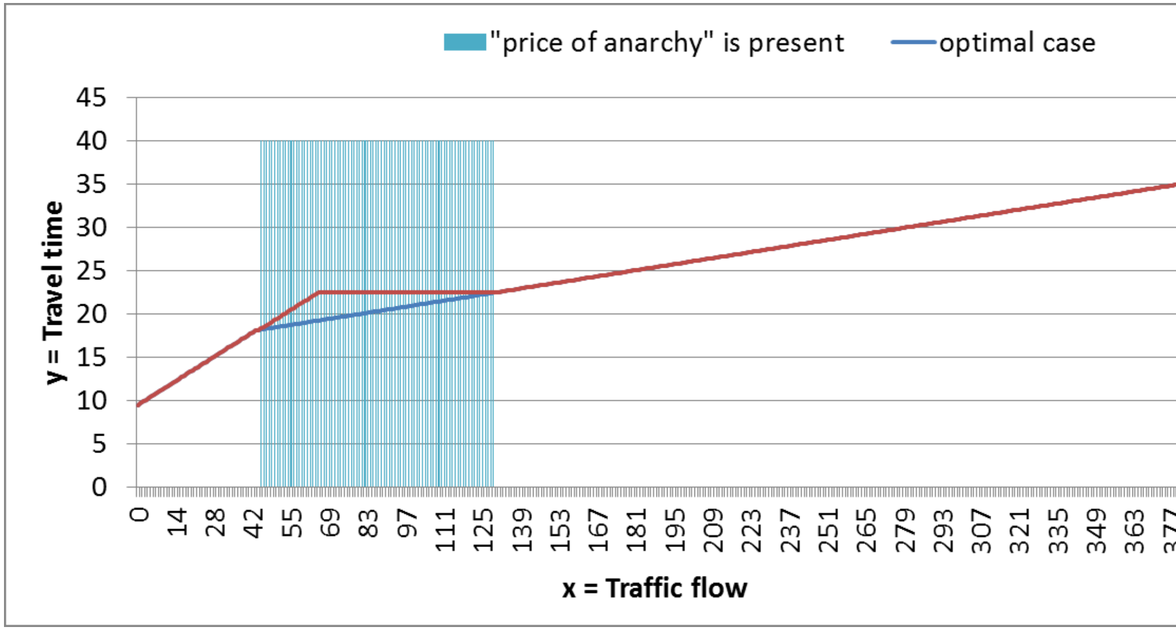
László Z. Varga. Benefit of Online Real-time Data in the Braess Paradox with Anticipatory Routing. In Samuel Kounev, Holger Giese, and Jie Liu, editors, 2016 IEEE International Conference on Autonomic Computing, ICAC 2016, Würzburg, Germany, July 17-22, 2016, pages 245–250. IEEE Computer Society, 2016. link

- the conjecture is the following: if simultaneous decision making is prevented in SNIP online routing games, then the benefit of online real-time data can be guaranteed
- first step:
  - SNIP online routing game over a Braess network
  - with a single incoming flow, and
  - we prove that in this game the worst case benefit of online real-time data is not more than 1+1÷225

GUARANTEED BENEFIT OF ONLINE REAL-TIME DATA IN THE ONLINE BRAESS PARADOX

- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$

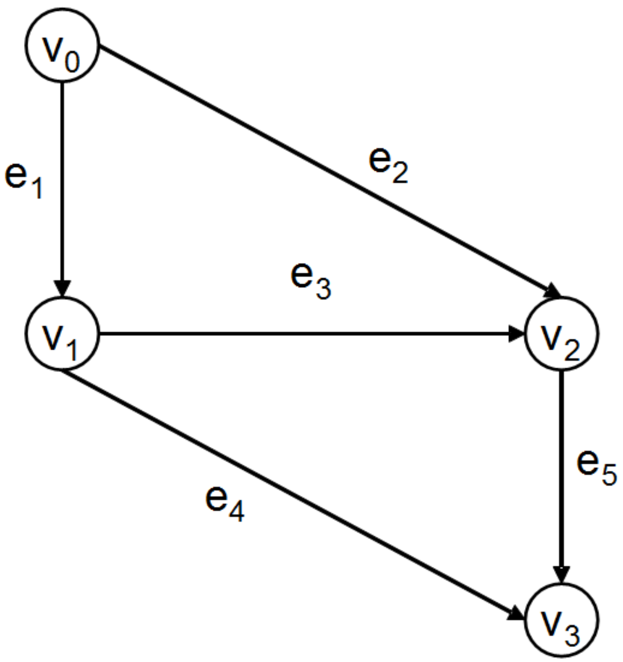




- the minimum following distance gap_e on all edges is 0.1

GUARANTEED BENEFIT OF ONLINE REAL-TIME DATA IN THE ONLINE BRAESS PARADOX

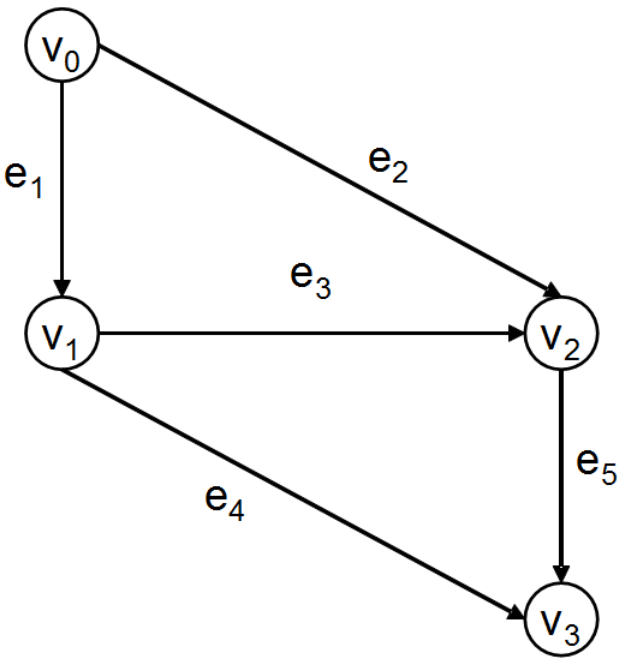
- c_{e1}=1+x÷10, c_{e2}=15, c_{e3}=7.5, c_{e4}=15, c_{e5}=1+x÷10



- Proposition 1.
  The actual travel time on edge e_1 is always the same as the predicted travel time
- Proposition 2.
  The actual arrival time at vertices v_1 and v_2 is always the same as the predicted arrival time

GUARANTEED BENEFIT OF ONLINE REAL-TIME DATA IN THE ONLINE BRAESS PARADOX

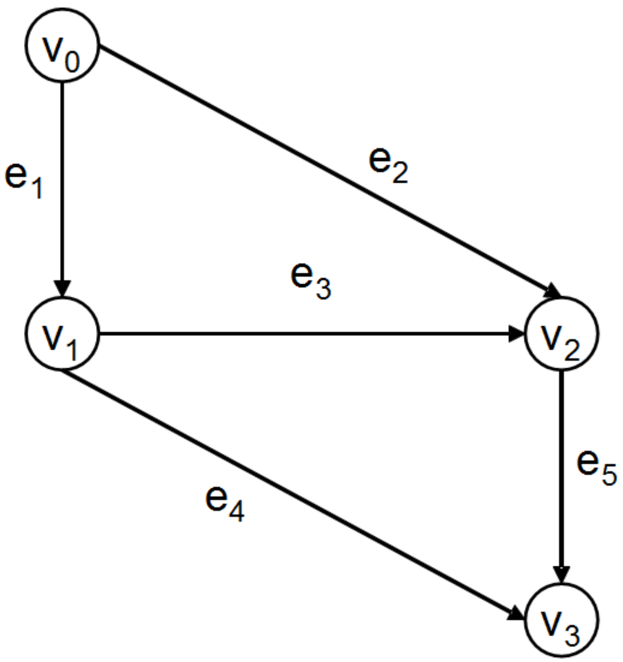
- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- Proposition 3.
  The actual arrival time at vertex v_3 on path $p_1=(e_1;e_4)$ is always the same as the predicted arrival time
- Proposition 4.
  If an agent selects the path $p_2=(e_1;e_3;e_5)$, then the predicted travel time on edge e_5 for this agent is less than or equal to 7.5

GUARANTEED BENEFIT OF ONLINE REAL-TIME DATA IN THE ONLINE BRAESS PARADOX

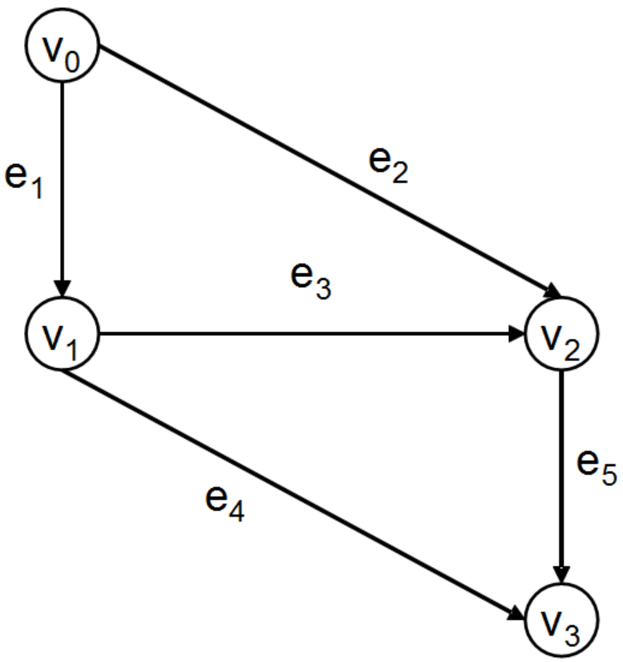
- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- Proposition 5.
  The actual and the predicted travel time on path $p_1=(e_1;e_4)$ is always less than or equal to $r_1\div10+16$
- Proposition 6.
  If $r_1<65$, then the agents always select the path $p_2=(e_1;e_3;e_5)$, and the travel time on p_2 is always less than or equal to $9.5 + 2*r_1\div10$

GUARANTEED BENEFIT OF ONLINE REAL-TIME DATA IN THE ONLINE BRAESS PARADOX

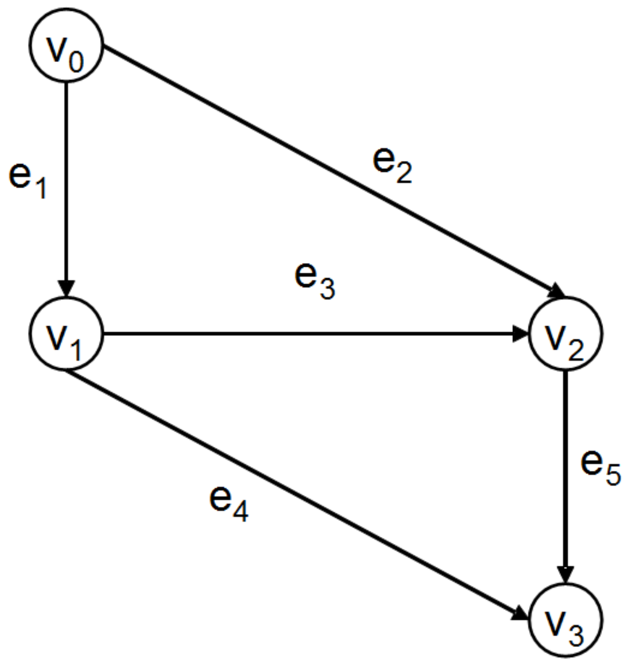
- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- Proposition 7.
  If $r_1\geq65$, then the travel time on path $p_3=(e_2;e_5)$ is always less than or equal to $r_1\div10+16.1$
- Proposition 8.
  If $r_1\geq65$, then the travel time on path $p_2=(e_1;e_3;e_5)$ is always less than or equal to $2*r_1\div10+9.6$

GUARANTEED BENEFIT OF ONLINE REAL-TIME DATA IN THE ONLINE BRAESS PARADOX

- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- the cost of non-online strategy: $2*r_1\div10 + 9.5$
- If $r_1<65$, then the worst case is $(2*r_1\div10 + 9.5)\div(2*r_1\div10 + 9.5)$
- If $r_1\geq65$, then the worst case on path $p_3=(e_2;e_5)$ is $(r_1\div10+16.1)\div(2*r_1\div10 + 9.5)$
- If $r_1\geq65$, then the worst case on path $p_2=(e_1;e_3;e_5)$ is $(2*r_1\div10+9.6)\div(2*r_1\div10 + 9.5)$

CONCLUSIONS

- a model for the metric and analysis of autonomous adaptation based on online real-time data
- it is known that autonomous self-adaptation with these strategies
  - sometimes make the system fluctuate,
  - sometimes some agents pay a price for the autonomous self-adaptation of the whole system,
- we have shown that intention-propagation-based prediction
  - is a form of cooperation, which helps to solve these problems
  - guarantees the worst case benefit of online data in the SNIP online routing game over the Braess network.

convergence to the equilibrium in intention aware online routing games

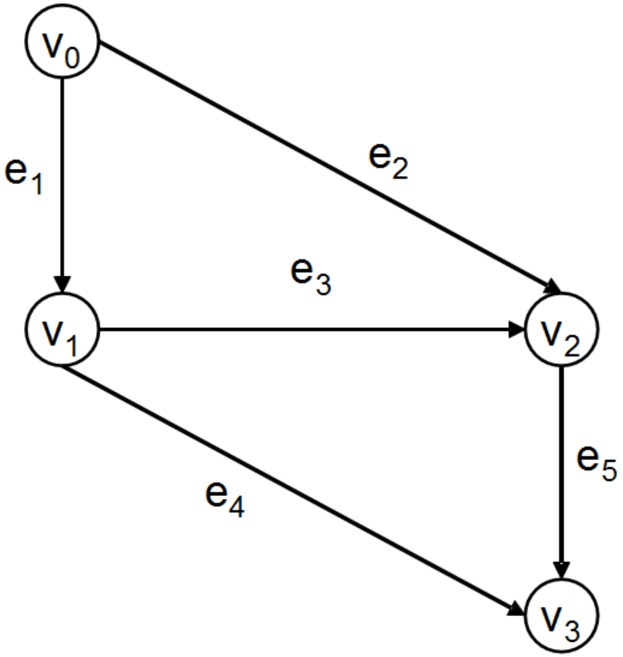
EQUILIBRIUM WITH PREDICTIVE ROUTING IN THE ONLINE BRAESS PARADOX

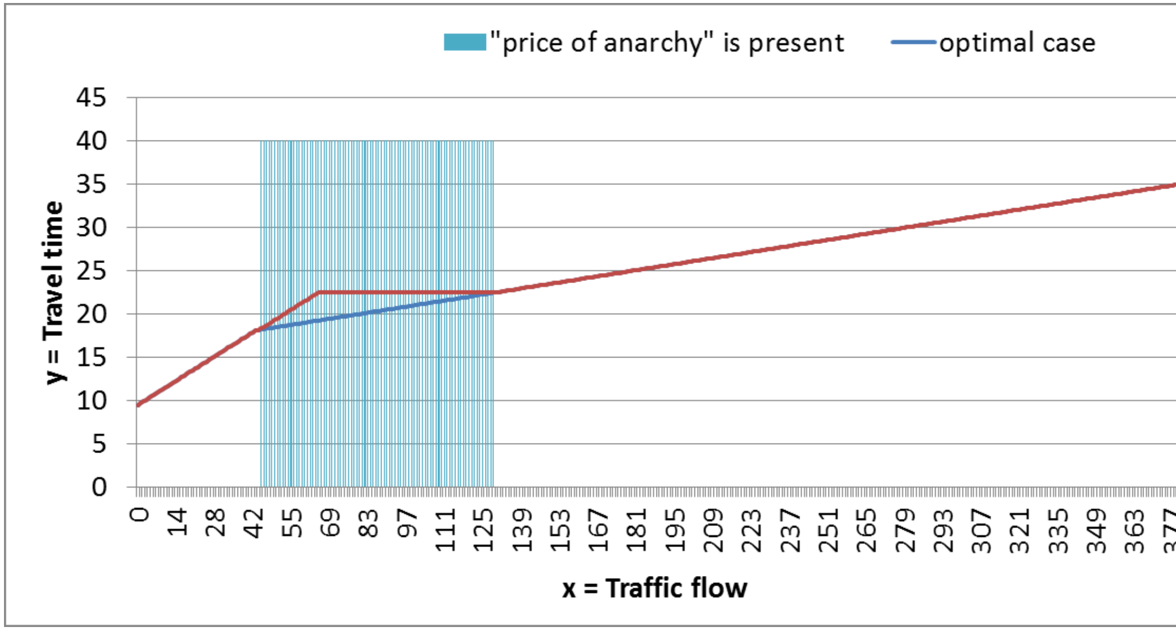
László Z. Varga. Equilibrium with Predictive Routeing in the Online Version of the Braess Paradox. IET Software, 11(4):165–170, August 2017. link

- the conjecture is the following: if simultaneous decision making is prevented in SNIP online routing games, then the system converges to the static equilibrium
  - there is a time limit after which the difference from the static equilibrium does not exceed a relatively small value
- first step:
  - SNIP online routing game over a Braess network
  - with a single incoming flow, and
  - we prove the convergence of this game

EQUILIBRIUM WITH PREDICTIVE ROUTING IN THE ONLINE BRAESS PARADOX

- $c_{e1}=1+x \div 10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x \div 10$

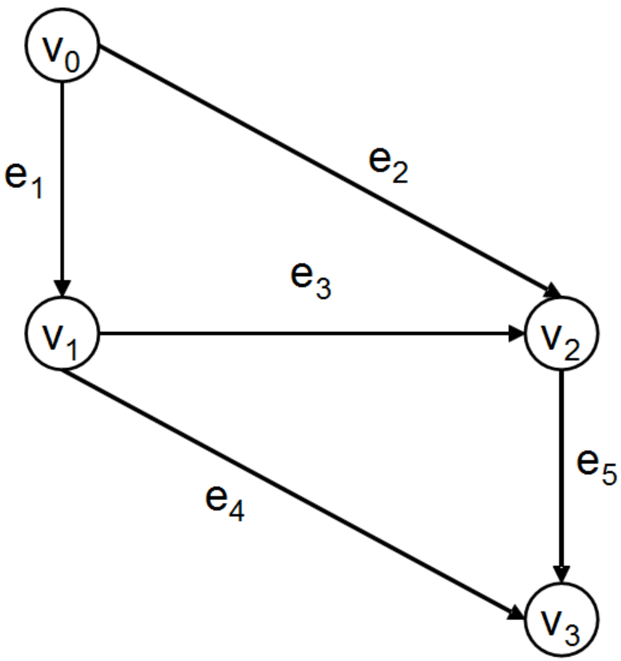




- the minimum following distance gap_e on all edges is 0.1
- propositions from previous slides hold

EQUILIBRIUM WITH PREDICTIVE ROUTING IN THE ONLINE BRAESS PARADOX

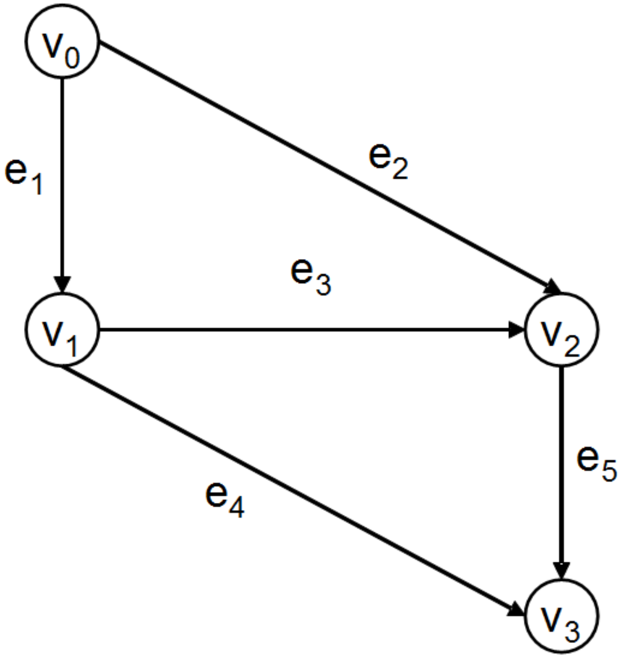
- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- Proposition 9.
  If the traffic flow on edge e_1 is below $r_1\div2$, then path $p_3=(e_2;e_5)$ is not selected
- Proposition 10.
  If $r_1\leq132$, then if the traffic flow on edge e_1 is below $r_1\div2$, then path $p_1=(e_1;e_4)$ or $p_2=(e_1;e_3;e_5)$ is selected and the actual travel time on these paths is not more than 22.6

EQUILIBRIUM WITH PREDICTIVE ROUTING IN THE ONLINE BRAESS PARADOX

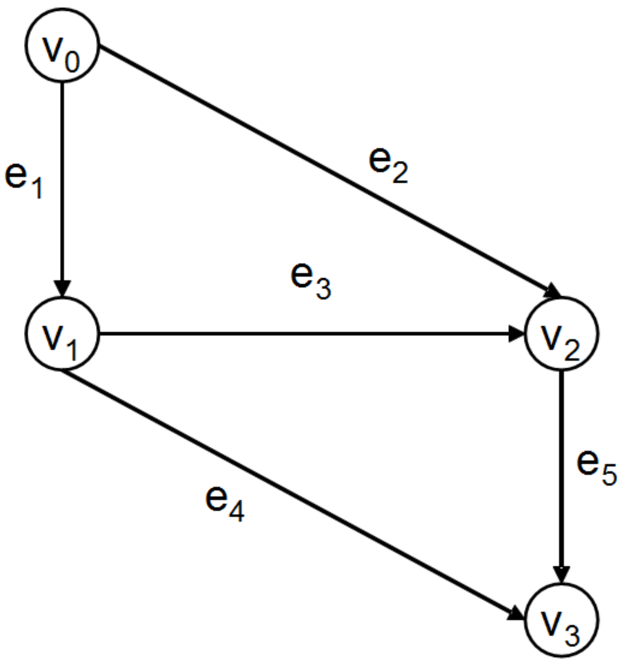
- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- Proposition 11.
  If $r_1 \leq 132$, then the traffic flow on edge e_5 is never above 66
- Proposition 12.
  If $r_1 \leq 132$, then the actual travel time on path $p_3=(e_2;e_5)$ is not more than 22.6

EQUILIBRIUM WITH PREDICTIVE ROUTING IN THE ONLINE BRAESS PARADOX

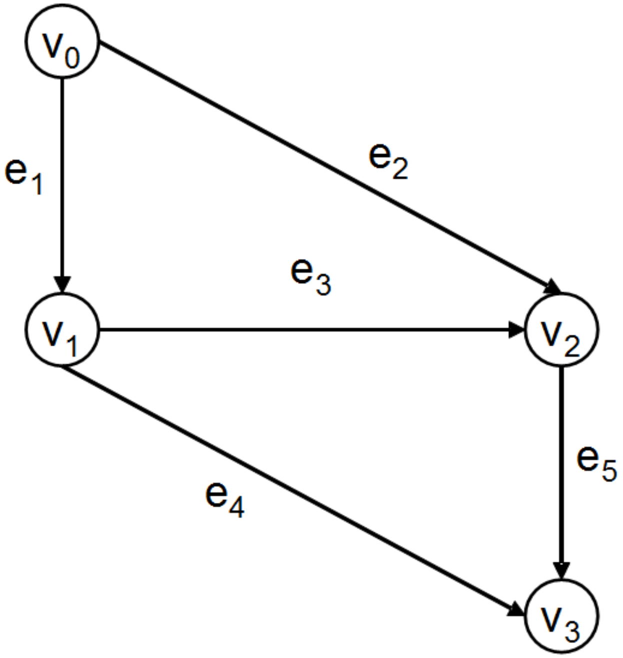
- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- Proposition 13.
  If $r_1 \leq 132$, then the actual travel time on path $p_1=(e_1;e_4)$ is not more than 22.6
- Proposition 14.
  If $r_1>132$, then if the traffic flow on edge e_1 drops below $r_1\div2$, then path $p_1=(e_1;e_4)$ is selected

EQUILIBRIUM WITH PREDICTIVE ROUTING IN THE ONLINE BRAESS PARADOX

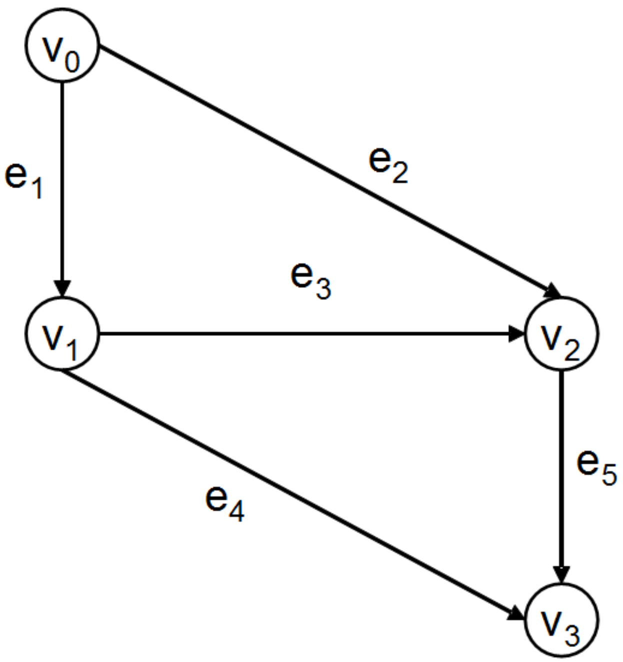
- $c_{e1}=1+x\div 10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div 10$



- Proposition 15.
  If $r_1>132$, then after some finite time the path $p_2=(e_1;e_3;e_5)$ will never be selected
- Proposition 16.
  If $r_1>132$, then after some finite time the traffic flow will alternate between paths $p_1=(e_1;e_4)$ and $p_3=(e_2;e_5)$

EQUILIBRIUM WITH PREDICTIVE ROUTING IN THE ONLINE BRAESS PARADOX

- $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=15$, $c_{e5}=1+x\div10$



- Proposition 17.
  If $r_1>132$, then after some finite time the traffic flow on edge e_1 will remain between $r_1\div2-1$ and $r_1\div2+1$, and the travel time on both $p_1=(e_1;e_4)$ and $p_3=(e_2;e_5)$ will be between $r_1\div20+15.9$ and $r_1\div20+16.1$
- Theorem.
  After some time, the travel times are at most the same as in the routing game model plus 0.1

CONCLUSIONS

- proved that the travel times in this online version of the Braess paradox is close to the Nash equilibrium of the classic Braess paradox
- there may be only a small additional travel time increase, which is due to the atomic nature of the traffic flows
- the coordination established by the intention-propagation-based prediction among the agents entering the network in a sequence is good enough to reduce the excessive swing of the system caused by the utilisation of real-time information

prediction methods in intention aware online routing games

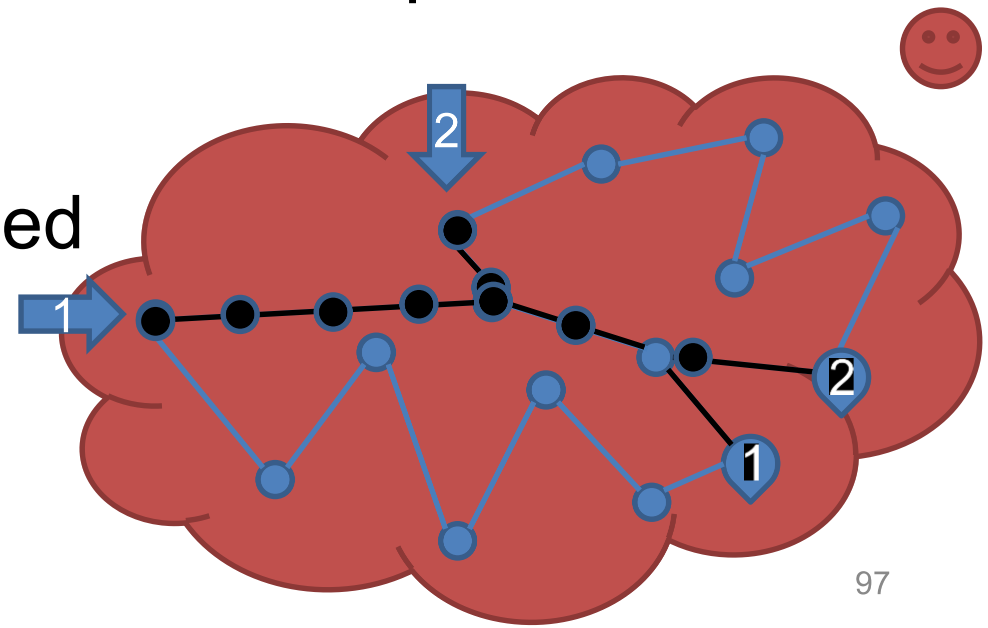
EQUILIBRIUM WITH DIFFERENT PREDICTION METHODS

László Z. Varga. Two Prediction Methods for Intention-aware Online Routing Games. In Proc. of 5th International Conference on Agreement Technologies, Évry, France, 14-15 December 2017, paper no.6. in the pre- proceedings. Springer-Verlag, 2017. link

- up to now, we assumed that the intention aggregation service can tell the future exactly
- we have seen that
  - „considerable" overtake may occur in non intention aware online routing games (see „single flow intensification")
  - „slight" overtake may occur in intention aware online routing games
- telling the exact future needs to know what the intention of the consequent agents will be in the future → complex computation
- simpler prediction methods are needed

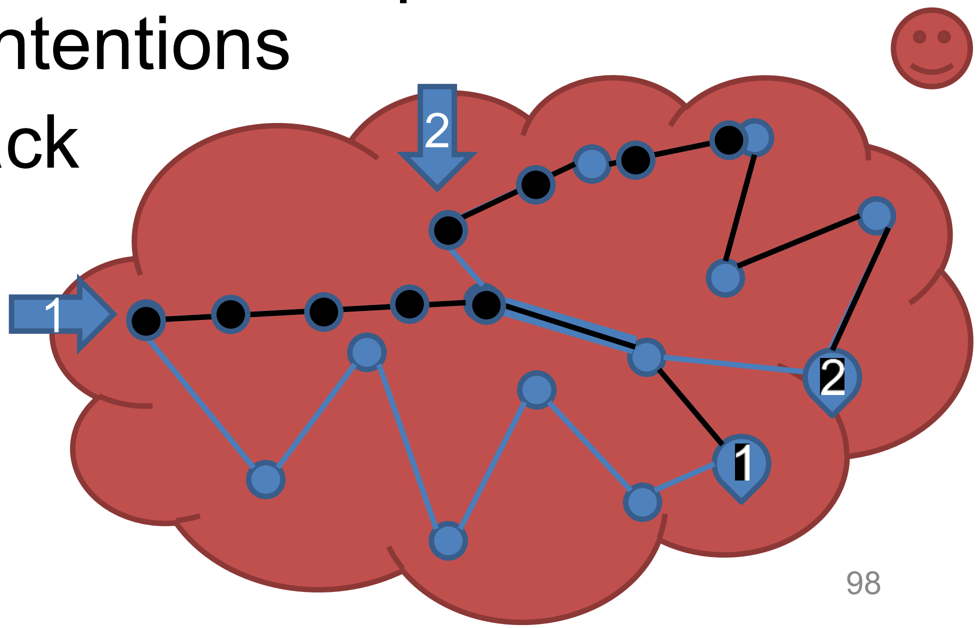
INTENTION-AWARE PREDICTION METHODS – DETAILED METHOD

- detailed prediction method
  - takes into account all the intentions already submitted to the central service,
  - then it computes what will happen in the future if the agents execute the plans assigned by these intentions,
  - the feedback is based on the predicted travel conditions

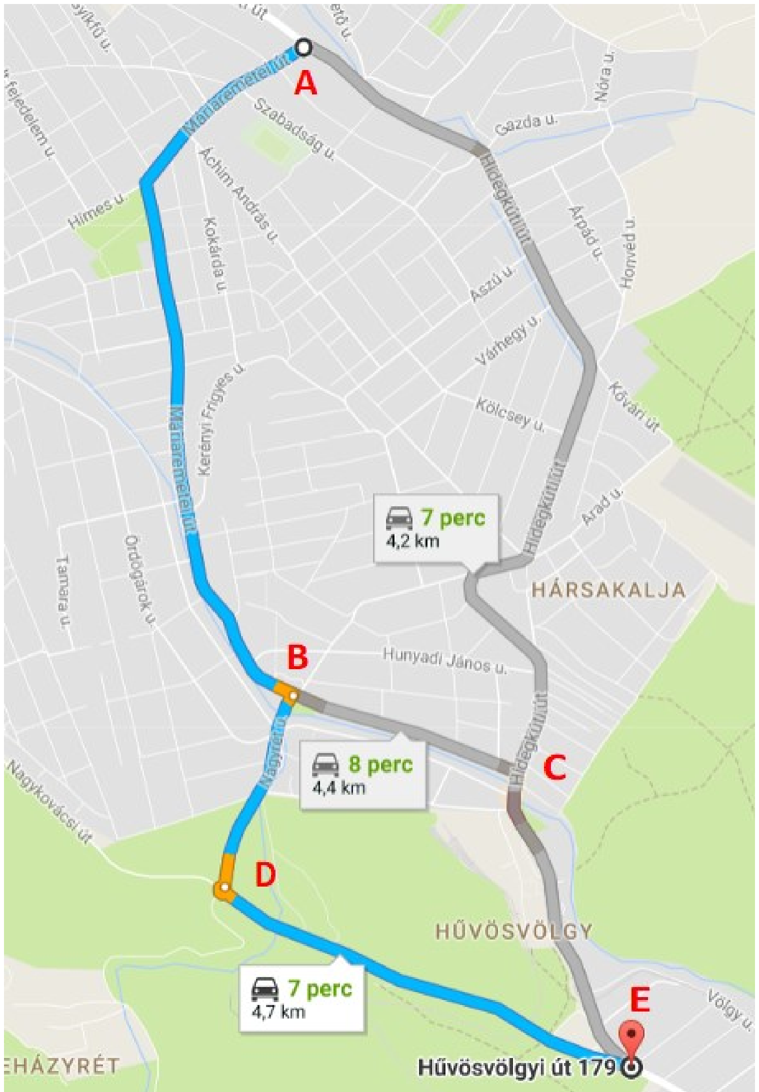


INTENTION-AWARE PREDICTION METHODS – SIMPLE METHOD

- simple prediction method
  - also takes into account all the intentions already submitted to the central service,
  - then it computes what will happen in the future if the agents execute the plans assigned by these intentions
  - however the feedback is based on the latest predictions for each road



EXPERIMENTAL SET-UP



$$c_{(A,B)} = 3.75 + 2.5 * flow \div 10$$
$$c_{(A,C)} = 4.65 + 3.1 * flow \div 10$$
$$c_{(B,C)} = 1.2 + 0.8 * flow \div 10$$
$$c_{(B,E)} = 3.3 + 2.2 * flow \div 10$$
$$c_{(C,E)} = 1.65 + 1.1 * flow \div 10$$

MEASUREMENTS

	Steady Flow						Pulsing Flow					
	No Pred.		Det. Pred.		Simple Pred.		No Pred.		Det. Pred.		Simple Pred.	
Flow	Max.	Avg.	Max.	Avg.	Max.	Avg.	Max.	Avg.	Max.	Avg.	Max.	Avg.
5	9.40	8.40	7.73	7.41	7.98	7.58	9.02	8.00	7.73	7.23	7.98	7.41
10	11.75	10.35	9.14	8.56	9.08	8.60	11.44	9.81	9.16	8.17	9.08	8.25
15	14.10	12.40	10.87	9.81	10.43	9.75	14.10	11.44	10.68	9.13	10.49	9.15
20	16.45	14.44	13.38	11.63	11.71	10.87	16.51	13.46	12.77	10.39	11.97	10.15
25	18.80	16.62	17.12	14.10	13.45	12.18	18.80	15.24	16.14	11.98	13.45	11.20
30	21.15	18.75	21.48	16.11	16.21	13.88	21.61	17.40	18.93	13.69	15.75	12.57
35	23.50	20.90	23.08	18.35	19.00	16.50	23.78	19.30	21.11	15.17	17.94	14.13
40	25.85	23.01	25.24	19.83	20.96	18.21	30.71	22.35	22.80	16.96	19.35	15.50
45	28.20	25.15	26.80	21.48	22.30	19.73	30.25	24.22	26.20	18.75	22.08	17.47
50	30.55	27.40	28.87	23.32	23.69	21.28	30.55	25.48	27.61	20.34	23.36	18.95

EVALUATION

- The goal of the experiments was to test the following hypotheses:

[H1:] Any of the above intention-aware predictive routing performs better than the non predictive routing.

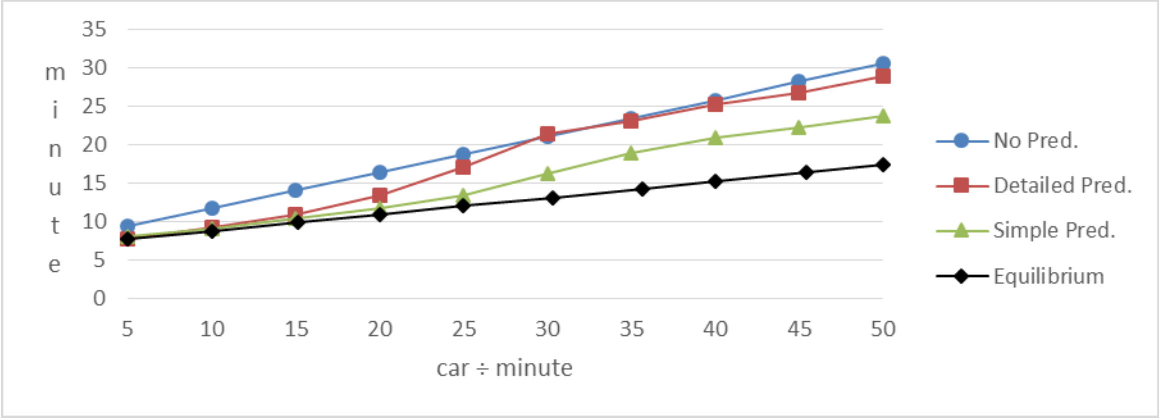
[H2:] The detailed prediction method performs better than the simple prediction method, because the detailed method gives more precise predictions.

[H3:] The intention-aware prediction methods limit the fluctuation of the congestions in the multi-agent system.

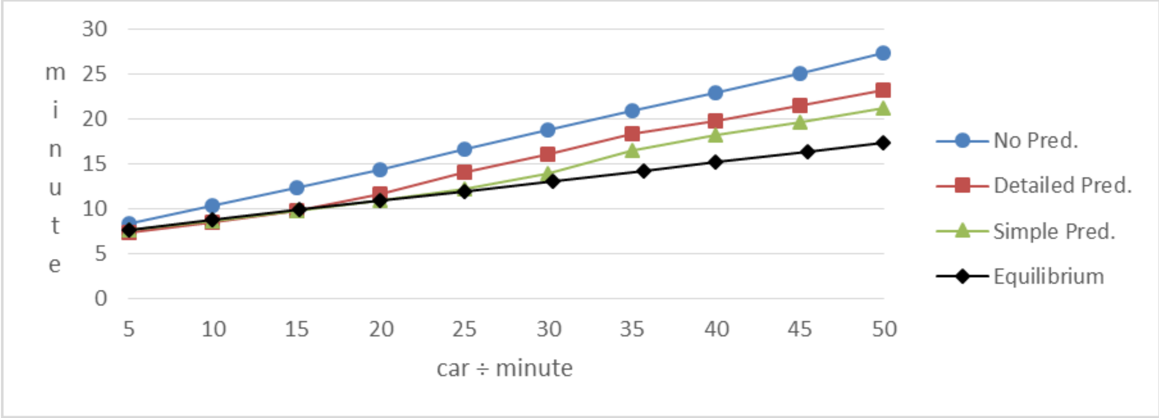
[H4:] The traffic converges to the equilibrium with the intention-aware prediction methods.

EVALUATION

- [H1:] Any of the above intention-aware predictive routing performs better than the non predictive routing. ✓
  – maximum travel times in the steady flow experiment

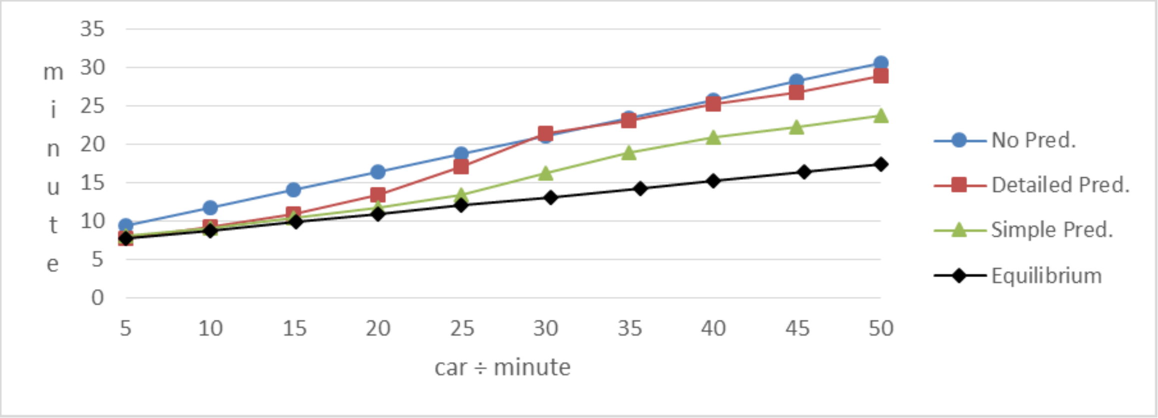


  – average travel times in the steady flow experiment

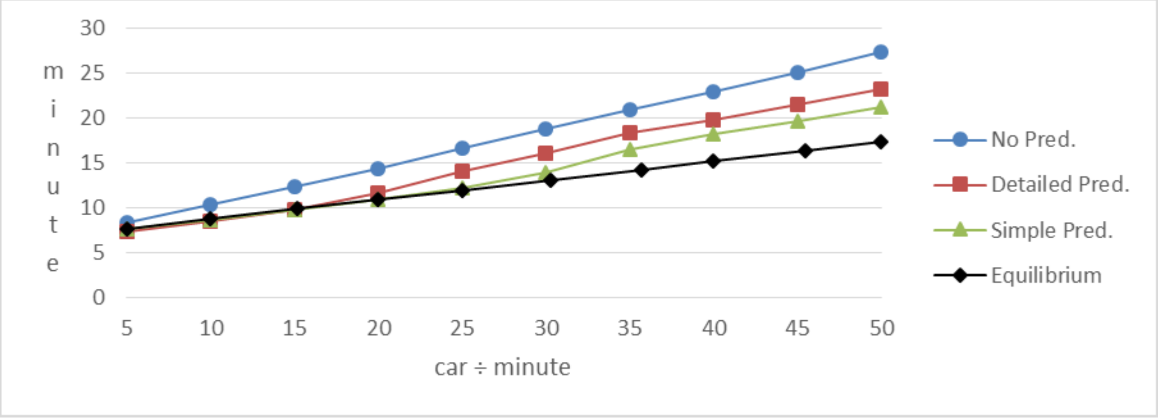


EVALUATION

- [H2:] The detailed prediction method performs better than the simple prediction method.
  - maximum travel times in the steady flow experiment



  - average travel times in the steady flow experiment



EVALUATION

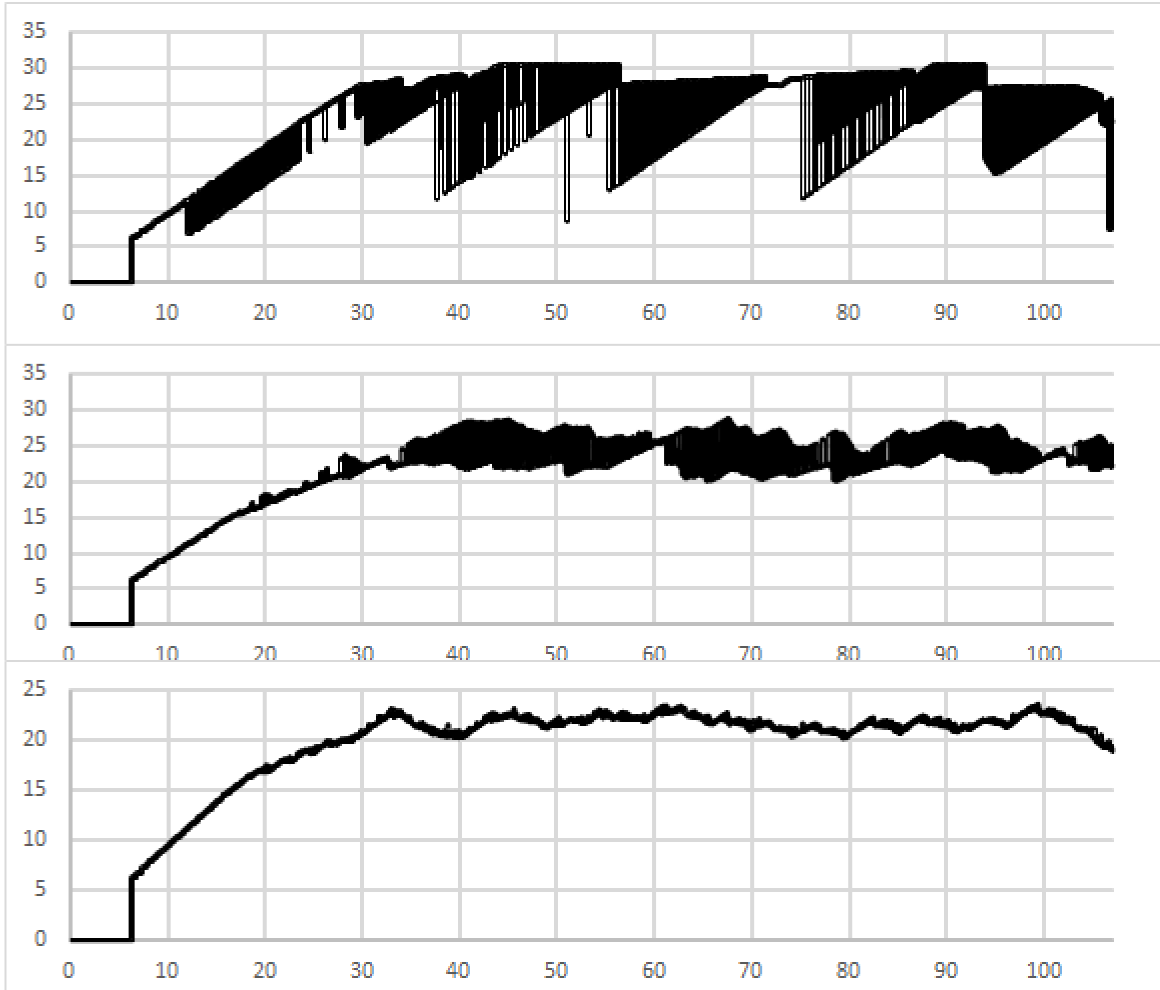
- [H3:] The intention-aware prediction methods limit the fluctuation of the congestions in the multi-agent system. ✓

steady flow, flow value 50

non predictive

detailed

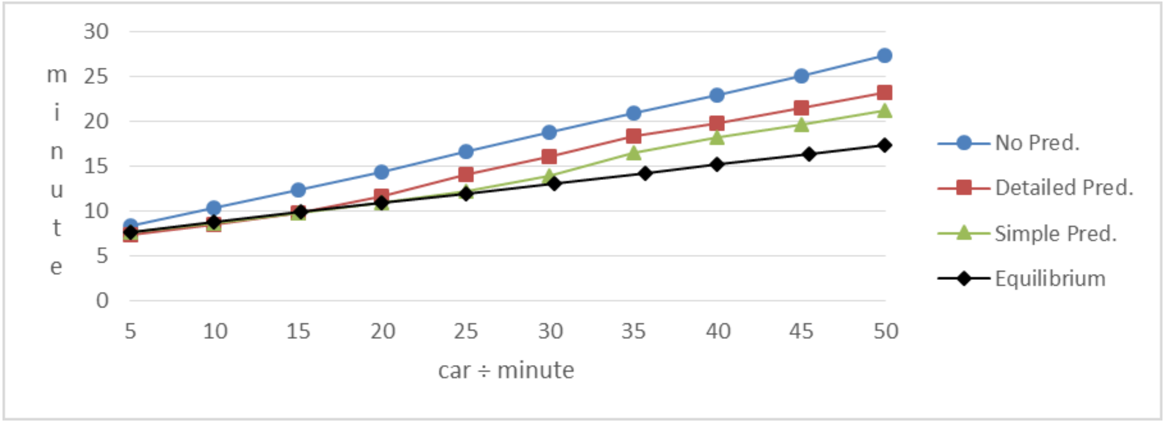
simple



EVALUATION

- [H4:] The traffic converges to the equilibrium with the intention-aware prediction methods. ?

- not fully confirmed, because the average travel times are higher than the equilibrium
- however we cannot expect that the equilibrium can be achieved exactly, because previous formal proof says that the travel times can come near to the equilibrium only within a threshold
  - average travel times in the steady flow experiment



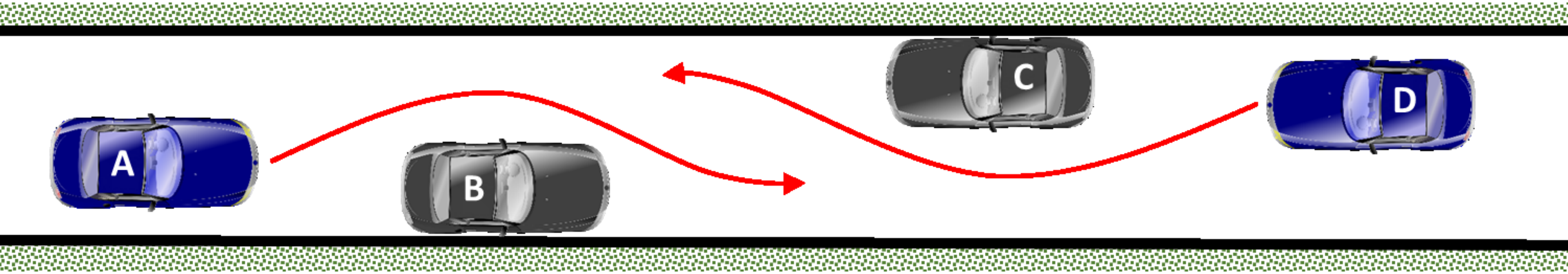
CONCLUSION

- we have defined two intention-aware prediction methods for online routing games and evaluated them in a real-world scenario
- the experiments confirmed that
  - the routing strategies using intention-aware prediction methods limit the fluctuation of congestions in online routing games,
  - and they make the system more or less converge to the equilibrium
- the convergence to the equilibrium needs further investigations
- unexpected result is that the simple prediction method performs better at higher traffic flow values than the detailed prediction method

Discussion – Autonomous Car Routing

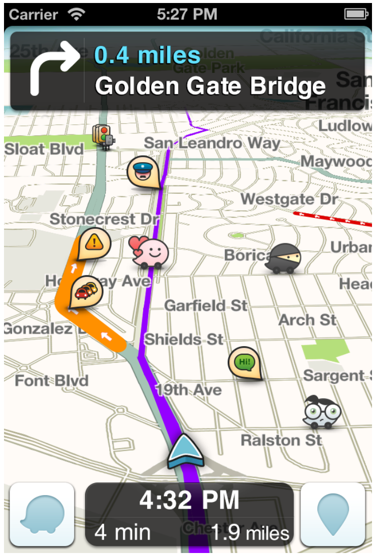
NON-COOPERATIVE GAMES: MINORITY/CHICKEN GAME

- agents who end up on the minority side win
- if every agent deterministically chooses the same action, then every agent is guaranteed to fail
- mixed strategy
  - model of diverse human behaviour, but not to control autonomous cars
- chicken game



COOPERATIVE DECENTRALISED AUTONOMOUS ADAPTATION

- coordination and communication is fostered by some kind of service of the infrastructure, but the control remains at the agents
- service / intention awareness
  - the agents send their intentions to the service
  - the service forecasts the future traffic situation based on the current traffic state and the intentions
  - the agents use this forecast to plan their trips
- the online navigation software like Google Maps and Waze know the intentions of the agents and could use this information to make predictions



INTENTION AWARE ONLINE ROUTING GAMES

László Z. Varga. How Good is Predictive Routing in the Online Version of the Braess Paradox? In ECAI 2016 - 22nd European Conference on Artificial Intelligence, 29 August-2 September 2016, The Hague, The Netherlands, volume 285 of Frontiers in Artificial Intelligence and Applications, pages 1696–1697. IOS Press, 2016. link

- if the agents try to maximise their predicted utility, then
  - in some networks and in some cases the traffic may be worse off by exploiting real-time information and prediction than without
  - there is no guarantee on the equilibrium
- however, it is verified that in a small but complex enough network, the agents might just slightly be worse off with real-time data and prediction
- currently the conjecture is that if simultaneous decision making among the agents is prevented, then intention aware prediction can make the traffic converge to the equilibrium in bigger networks as well

CENTRALISED ADAPTATION

- central control authority for geographical territories
- autonomous car checks in at the control authority, and the control authority would tell the exact route to follow to the destination
- similar to the operation of airports
- the autonomous car becomes something similar to a remote controlled car
- the users of the autonomous cars may not accept this concept

DISCUSSION

- autonomous cars → optimal traffic ?
- decentralised autonomous adaptation
  - cannot be verified if the agents autonomously follow their preferences to adapt to their environment
- autonomous adaptation using intention aware prediction
  - there is no guarantee on the equilibrium, although in some conditions there is hope
- centralised adaptation
  - feeling of a remote controlled car ?
- else …

THANK YOU FOR YOUR ATTENTION!

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