Wavelet design for analog implementation

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Wavelet transforms are nowadays used extensively in the field of signal processing. A common issue that practitioners must deal with is the question of which wavelet to use. A good choice depends both on the application and on the morphology of the signal. One approach is to employ a limited number of standard wavelets from one of the many toolboxes that are available. Properties such as vanishing moments, filter length, time-frequency resolution, etc., can be taken into account. One simply selects the wavelet with the best performance in the setting of the given application. Another approach is to *design* a wavelet for the application at hand. This requires a framework which consists of a parameterized class of wavelets and a criterion to measure the performance of a wavelet. Here, a methodology is proposed to design and implement wavelets for the *analog* domain, which builds on previous research in [4] and [3].

The theory of wavelets derived from filter banks provides a well understood framework for *digital* signal processing, using FIR filters. Important aspects are: (i) perfect reconstruction, (ii) orthogonality of the filter bank and the underlying wavelet based multi-resolution structure, (iii) flatness of the filters and vanishing moments in the wavelets, (iv) smoothness of the wavelets. Efficient implementation of such filter banks can be achieved with polyphase filters and a corresponding lattice structure. In [1, 2] it is discussed how to parameterize the class of orthogonal wavelets which involve FIR filter banks of order $2n-1$. The filter coefficients $h_0, \ldots, h_{2n-1}$ are reparameterized using n parameters $\theta_0, \ldots, \theta_{n-1}$ which can be chosen freely to obtain an orthogonal filter bank. In [4] constraints are given which ensure a first and a second vanishing moment.

In [4] the question is addressed of what is a good criterion for wavelet design. It is argued that in the *digital* domain for denoising and compression purposes, 'sparsity' of the wavelet representation of a prototype signal is important. There it is shown that sparsity can be achieved through the principle of 'maximization of variance'. In the present framework it holds that maximization of the variance of the absolute values of the wavelet coefficients corresponds to L_1-minimization, while maximization of the variance of the corresponding energies (i.e., the squared wavelet coefficients) corresponds to L_4-maximization. By optimizing the wavelet representation of a prototype signal with respect to either one of these criteria, orthogonal wavelet design can be conducted. Experiments in [4] indicate that L_4-maximization performs best and is to be preferred.

The wavelets obtained by the procedure above can be implemented in good approximation in *analog* circuits in the following way. First the underlying (continuous) wavelet function is calculated from the filter coefficients using the iteration scheme from [1]. Then the wavelet function is time-reversed and time-shifted and approximated by the impulse response of a continuous-time linear system using L_2-approximation, see [3]. The wavelets may not be smooth, which can complicate the approximation. Finally, the approximating linear system is implemented in the analog domain with dynamic translinear circuits (for various scales). In this research it is investigated whether the proposed approach leads to wavelets with an improved performance in the area of cardiac signal processing, when compared to the performance achieved by standard wavelets such as the Gaussian wavelet and the Morlet wavelet.

References

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