L_2-Approximation of Wavelet Functions

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1 Introduction

Computations in the analog domain are significantly less power consuming than computations in the digital domain, mainly due to the power consumption of A/D-converters. It is therefore opportunistic to perform computations in the analog domain for applications where power consumption is a critical issue. Wavelet transformations $W(t, \sigma)$ can be performed in the analog domain by approximating them with linear systems (LS). In earlier work [1] this was accomplished with Padé-approximation of wavelet functions for implementation in dynamic translinear (DTL) circuits that are current- instead of voltage-based which offers a number of advantages [2].

2 Approximation of wavelets for linear systems

A continuous-time wavelet transform is the L_2-inner product of a wavelet $\psi(t)$ with a signal $f(t)$. This type of transform can be implemented with an LS since if a time signal $f(t)$ is passed through an LS, then $f(t)$ is convoluted with the impulse response $h(t)$ of that LS. As a result a wavelet transform can be approximated by an LS of finite order by approximating the time-reversed wavelet function $\widehat{\psi}(t)$ with the impulse response $h(t)$ of the LS. Only the implementation of (strictly) proper rational functions is feasible and therefore the wavelet should be shifted in time to avoid truncation of energy such that a time-reversed and shifted wavelet $\widetilde{\psi}(t)$ is approximated with impulse response $h(t)$.

3 L_2-approximation of wavelets

L_2-approximation theory provides a framework for studying the problem of wavelet approximation, which offers a number of advantages: firstly, it is quite appropriate to use the L_2-norm to measure the quality of an approximation $h(t)$ of the function $\widetilde{\psi}(t)$ since $W(t, \sigma)$ is an L_2-inner product. Secondly, it is desirable that the approximation $h(t)$ of $\widetilde{\psi}(t)$ behaves equally well for all time instances t. As a third advantage L_2-approximation allows for a description in the time domain as well as in the Laplace domain.

For the generic situation of stable systems with distinct poles, the impulse response function $h(t)$ is a linear combination of damped exponentials and exponentially damped harmonics. This makes it possible for low order systems, to propose an explicitly parameterized class of impulse response functions among which to search for a good approximation of $\widetilde{\psi}(t)$, as discussed in [3].

4 Obtaining a good starting point

In order to approximate a wavelet function with the L_2-approach a starting point is required. The choice of this point is significant due to the existence of local optima. To obtain a good starting point for the L_2-approximation approach, one can start by constructing a high-order model and applying model reduction with for example the balance and truncate method, so that an initial model with the appropriate order n is obtained. To do that we start with a sampled version of the time-reversed and shifted wavelet $\widetilde{\psi}(t)$, from which a discrete-time MA-model is created, that is reduced, converted to continuous time and is further reduced such that it has the required order n as described in [4].

References

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